GET-2230B

Power System Protection

USE OF R-X DIAGRAM

USE OF THE R-X DIAGRAM IN RELAY WORK

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GENERAL ELECTRIC

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THE USE OF THE R-X DIAGRAM IN RELAY WORK

INTRODUCTION

Practical system economics demand heavier line loadings than were considered possible years ago. These increased loadings have focused attention on system stability and its horde of associated problems. The consequent emphasis on circuit reliability means that more and more dependence must be placed on the system relaying. Because of this increased dependence, we demand increased exactness in performance. Simple relays, actuated by a single operating quantity, are found lacking when measured by these newer performance requirements. Thus, we have turned more and more to relays actuated by multiple operating quantities. For instance, we find that simple overcurrent relays are forced to yield more and more of the system relaying responsibility to some form of ohmic relaying; that is, relaying actuated by three electrical quantities : voltage, current and phase angle. Our older relay application techniques based on the overcurrent concept are gradually being replaced by newer techniques based on the complex relationship between voltage and current. The application tool for these ohmic relays is the R-X diagram. Before we examine this tool, however, a brief review of relay requirements and relay characteristics is highly desirable.

For the sake of simplicity and brevity, we will confine this discussion to line relays and balanced three-phase conditions.

The line relay has the primary responsibility of closing the line circuit-breaker trip circuit promptly for any fault within the limits of the line. As we all know, this part of its job is simplicity itself when compared to its responsibility for not tripping unless a fault actually exists and lies between the line terminals. 'This latter part of the task is the part that is responsible for nearly all of the grey hairs in any relay department.

As an illustration, let us suppose that we were given the task of writing the major commandments for an ideal relay. We would find that they would fall naturally in two groups as follows:

"Thou Shalt-"

1. Thou shalt trip all faults within thy zone of protection irrespective of changes in generation.
2. Thou shalt trip these faults at highest speed ; yea, thou shalt make thy tripping decisions in terms of a second split into a hundred parts.

"Thou Shalt Not-"

1. Thou shalt not trip faults outside thy zone of protection except in back-up assistance to a failing brother.
2. Thou shalt not trip under heavy load conditions even though thy coils do carry much current.
3. Thou shalt not trip during power swings, denying always the tempting surges of current and voltage.

It is easy to see why any relay would have difficulty in keeping these commandments. As a matter of fact, these requirements have eliminated the over-current relay on important lines. The severe requirements of these. commandments have thus forced the development and utilization of the ohmic relays.

OHMIC ELEMENTS ON THE R-X DIAGRAM

What is an ohmic relay? We have broadly justified the name, "ohmic," by the statement that these relays operate in response to the three variables: voltage, current and phase angle. In order to appreciate this name, and in order to understand the operation of these relays we must consider the various relay elements individnally.

In general, these elements respond to at least three of the four familiar torque-producing components :

1. Voltage Component (Torque proportional to $\mathbf{E^2}$)
2. Current Component (Torque proportional to $\mathbf{I^2}$)
3. Product Component [Torque proportional to $\mathbf{E \times I \times f(\theta)}$]
4. Control Spring Torque

Thus, we can write a general torque equation for an ohmic element :

$$\text{Torque} = \pm K_1 E^2 \pm K_2 I^2 \pm K_3 \text{ EI } f(\gamma,\theta) \pm K_4$$

The conventions which we have adopted for this equation are :

(a) contact-closing torque is positive; (b) K_1, K_2, K_3 are independent design constants which may be used with either sign and varied in magnitude to meet requirements ; (c) K_4 represents the spring torque, assumed to be constant ; (d) γ is the design angle of maximum torque ; (e) E, I, and θ are the familiar operating quantities supplied to the relay. (γ and θ are angles by which I lags E.)

As the first example of the use of this equation, we will select $K_1 = K_2 = 0$, K_4 negligible, K_3 positive and $f(\gamma,\theta) = \sin(90° + \gamma - \theta)$. The equation now reads : $T = + K_3 \text{ EI } \sin(150° - \theta)$ for a relay where $\gamma = 60°$. This is recognizable as a directional element, wherein positive, contact-closing, torque is realized for values of θ between 330° and 150°, with maximum positive torque at $\theta = 60°$.

For our second example, we will choose $K_1 = 0$, K_4 negligible, K_2 positive, $f(\gamma,\theta) = \sin\theta$, and use

the minus sign for K_3. Our equation becomes : $T = + K_2I^2 - K_3EI \sin \theta$. In order to analyze this equation quickly, we will substitute (E/I) sin $\theta = X$, thusly ; $T = + K_2I^2 - K_3I^2X$. This is the equation for a reactance element which will operate to close its contacts whenever X drops below a value determined by the proportion of K_2 to K_3.

The next example will be the impedance element wherein we, as designers, will select K_1 as negative, K_2 positive, K_3 as zero and K_4 as negligible. Our equation: $T = -K_1E^2 + K_2I^2$. Once again we will substitute for ready analysis another relationship, E/I = Z. Rearranging the above equation: $T = +K_2I^2 - K_1I^2Z^2$. Thus the element will operate to close its contacts whenever Z drops below a value determined by K_2 and K_1.

The last element we will spend time to discuss might be called a directional element with voltage restraint. Selecting $K_2 = 0$, K_4 negligible, a minus sign for K_1 and $f(\gamma,\theta) = \sin (90° + \gamma - \theta)$, with K_3 positive, we have an equation $T = +K_3EI \sin (90° + \gamma - \theta) - K_1E^2$. If we use the relationship E/I = Z to simplify this equation we find: $T = K_3I^2Z \sin (90° + \gamma - \theta) - K_1I^2Z^2$. This relay will operate whenever K_1Z^2 is less than $K_3Z \sin (90° + \gamma - \theta)$.

This is very interesting as far as we've gone, but how do we evaluate these operating characteristics ? Let's examine the last element described by $T = K_3EI \sin (90° + \gamma - \theta) - K_1E^2$. Positive, contact-closing torque will be realized whenever $K_3EI \sin (90° + \gamma - \theta)$ exceed K_1E^2; negative, or contact-opening, torque will be realized whenever the reverse is true. The boundary of relay operation is thus defined by T = 0, i.e., when operating (positive) torque equals restraint (negative) torque. With this in mind, we can plot I sin $(150° - \theta) = (K_1/K_3)$ E (for $\gamma = 60°$) for a single, definite value of voltage, E = E,, as shown in Fig. la. If we select other values of voltage, we must draw new curves as shown on Fig. lb for $E = 0.5E_s, 1.0E_s, 2.0E_s$. Thus we have arrived at an I vs θ plot of this element's operating characteristic, using E_s as a parameter.

Again, we are tempted to say that this is very interesting, but how can we apply such a relay since the relay voltage will be different for different system faults and, more confusing, since it will be different for the same fault under different system conditions ?

The answer to this question will be found in the R-X diagram. The value of the R-X diagram lies in the two facts :

1. Ohmic relay characteristics can be simply shown. This is true because these characteristics may be plotted in terms of only two variables, R and X (or Z and θ), rather than the three variables, E, I and 0.

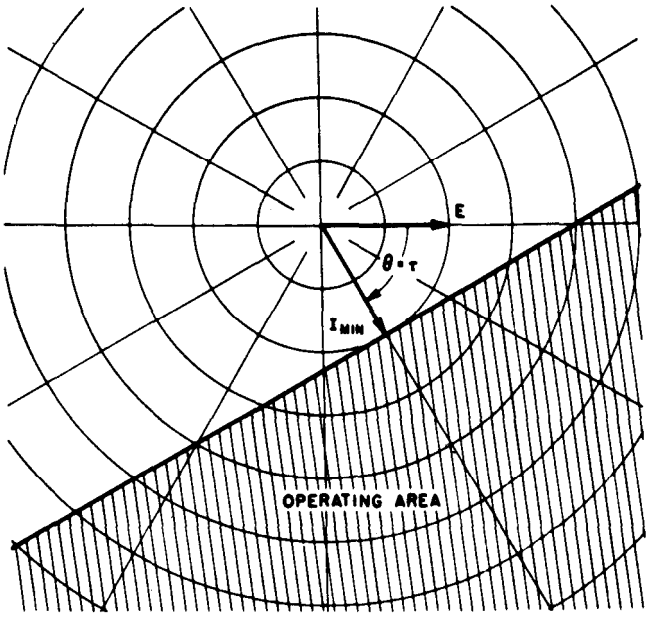


Fig. 1a. Operating Characteristic

$EI \sin (150 - \theta) = \frac{K_1}{K_3} E^2$

E = Constant
(60" MHO Element)

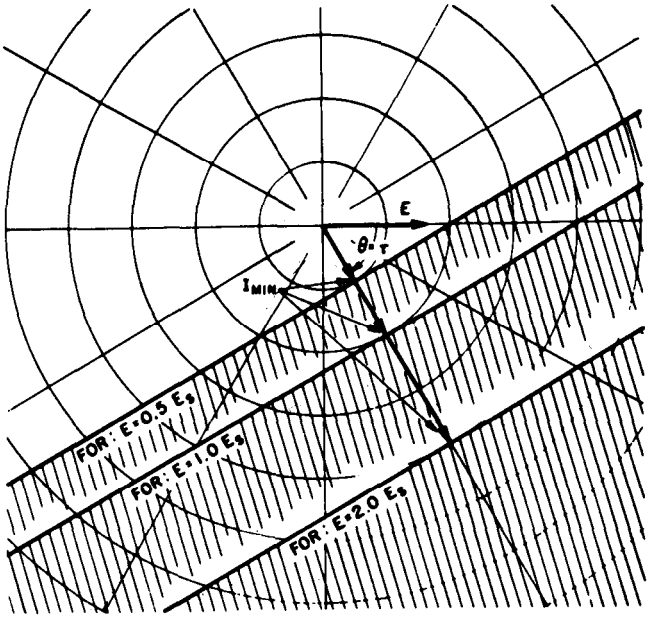


Fig. 1b. Operating Characteristic

$EI \sin (150 - \theta) = \frac{K_1}{K_3} E^2$

E = 0.5 E,, 1.0 Es, & 2.0 Es
(60° MHO Element)

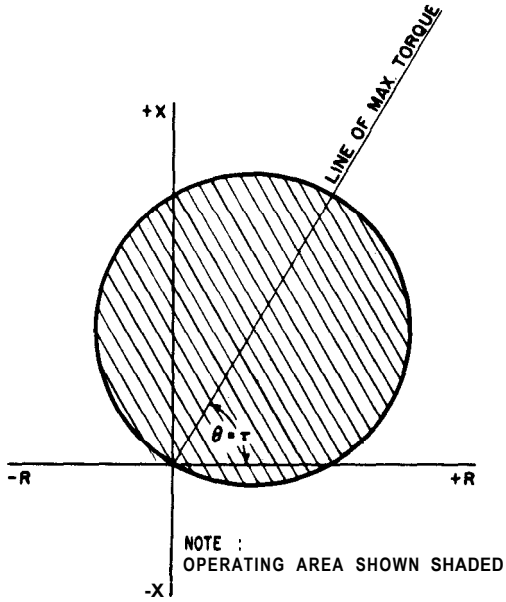


Fig. 2. Operating Characteristic

$$Z = \frac{K_3}{K_1} \text{ Sin } (150 - \theta)$$

(60° MHO Element)

2. System conditions affecting the operation of these relays can be shown on the same diagram. These points will be covered separately and then considered together.

OHMIC RELAY CHARACTERISTICS ON THE R-X DIAGRAM

Consider again the equation used for Fig. 1: I sin $(150° - \theta) = (K_1/K_3)$ E. If we recognize the fact that the impedance to a fault from the relay location is shown to the relay by the relationship E/I = Z, this equation becomes (K_3/K_1) sin $(150° - \theta) =$ Z. This equation defines the operating point of the relay in terms of Z and may be plotted on an R-X diagram as shown in Fig. 2. Positive contact-closing torque will be realized whenever (K_3/K_1) sin $(150° - \theta)$ is greater than Z, that is, whenever the fault impedance falls inside the circle. This simple plot does not depend on any parameters of operating quantities but defines the operating characteristic for all values of E, I and θ. This is the characteristic of the mho element used in the GCY relay.

In a similar manner, we can simplify the equations for the impedance element, plotted in Fig. 3, and the reactance element, plotted in Fig. 4. All values of fault impedance which will cause the relay to close its contacts lie in the shaded areas of these curves. It will be observed that the elements of Fig. 3 and 4 operate similarly to that of Fig. 2, in that a restraint torque and an operating torque just balance at the changeover point from positive

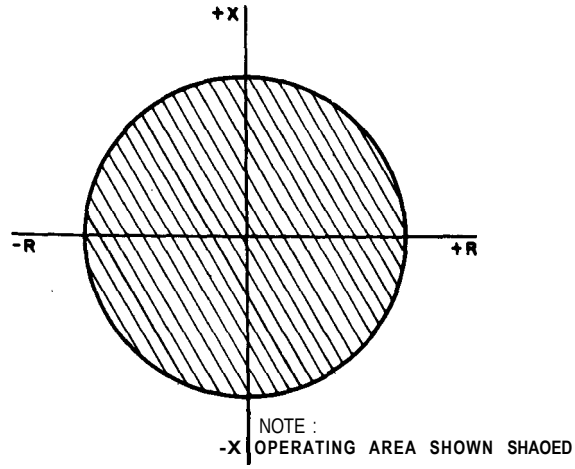


Fig. 3. Operating Characteristic

$$Z = \sqrt{\frac{K_2}{K_1}}$$

(Impedance Element)

to negative net torque. The mathematical treatment of these relay elements is therefore similar. The directional element is an exception as shown by the equation: $T = +K_3EI \sin (150° - \theta)$. From this equation, we see that a torque is developed for all values of E and I except for two values of θ: 150° and 330°. The sign of this torque will be positive for values of θ from 330" to 0° to 150° and negative for values of θ from 150° to 270° to 330°. Maximum torques will be developed at 60°

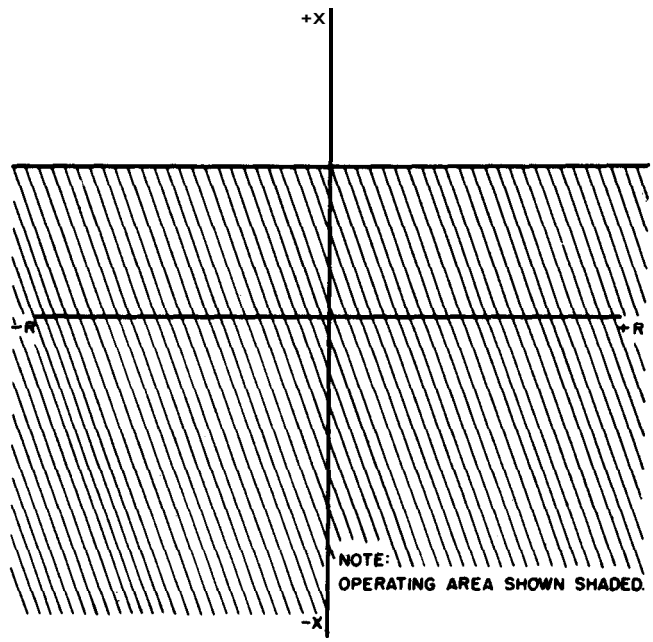


Fig. 4. Operating *Characteristic*

$$X = \frac{K_2}{K_3}$$

(Reactance Element)

(positive torque) and 240" (negative torque). Thus, the operating line of the relay may be drawn as shown on Fig. 5 if the control spring is neglected. It should be noted that the directional element is not an ohmic element in the usual sense, although its characteristic may be conveniently shown on an R-X diagram.

Another useful characteristic is found in the "offset mho" element. This element is similar to the mho element but has a portion of the operating current introduced in the voltages of the equation. Changing the fraction of the operating current thus used will change the offset as shown in Fig. 6. (In the practical relay, this offset is determined by taps.)

This completes the list of ohmic elements which will be described, although there are many other interesting and useful variations of the principles covered above. Our next concern is the combination of these elements to form a distance relay. This discussion will be confined to three types of distance relays and a brief review of out-of-step blocking. All three of the distance relays will be assumed to have an ideal time-distance characteristic as shown in Fig. 7. The 1st zone gives instantaneous operation while the 2nd and 3rd zones complete the trip circuit only through the contacts of a timer to give time delay tripping as shown.

(1) The Impedance Relay is composed of three impedance elements, each adjusted for a different

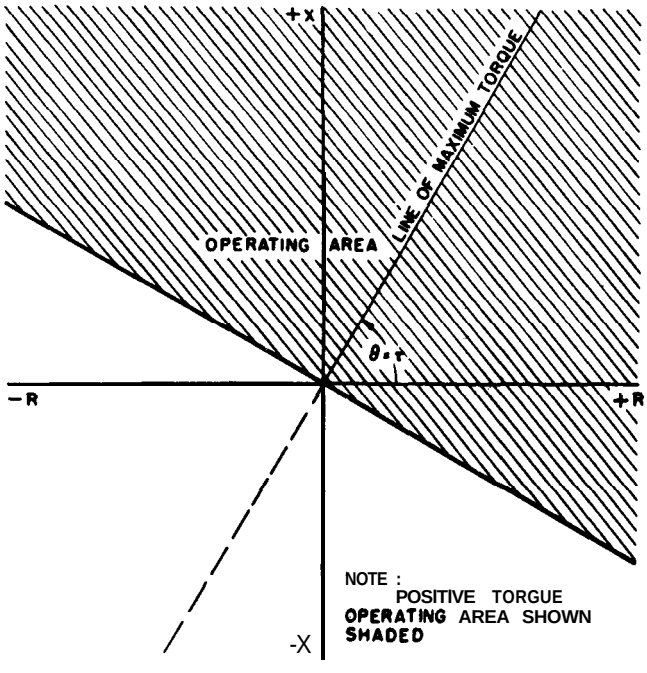


Fig. 5. Operating Characferisfic
$T = +K_3 EI \sin(150 - \theta)$
(Directional Element)

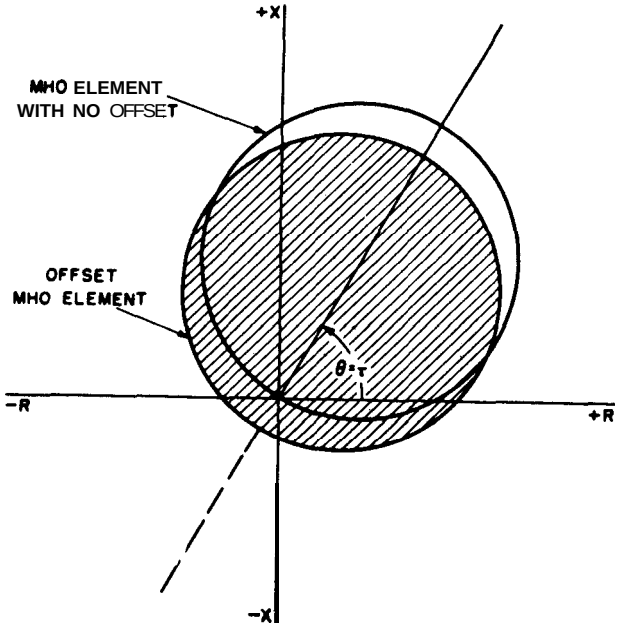


Fig. 6. Operating Characteristics of MHO *Element* And of Offset MHO Element

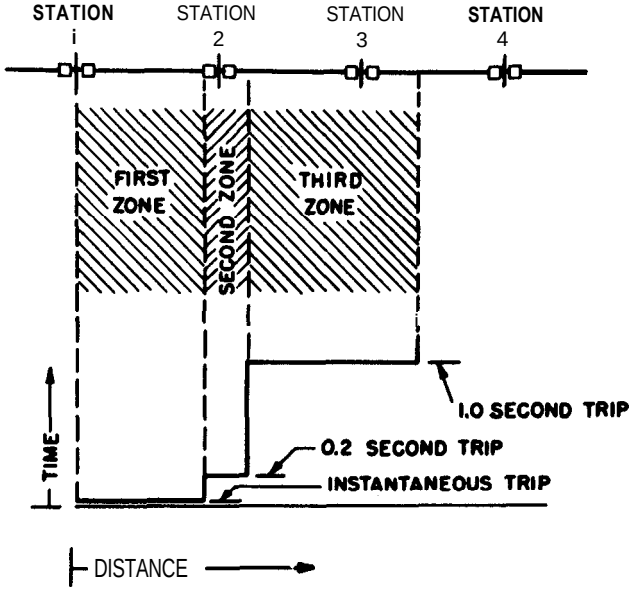


Fig. 7. Illustration of Time-Distance Plot
For Ideal Single Circuit Lint Shown
Relay Location: Station 1

ohmic reach. The directional element controls the tripping circuits for all three elements thus preventing relay operation for faults in the backwards direction. The characteristics of this relay are shown in Fig. 8.

(2) The Reactance Relay is composed of a reactance element, which gives 1st and 2nd zone protection, and a mho element which doubles as the starting unit and the 3rd zone protection element.

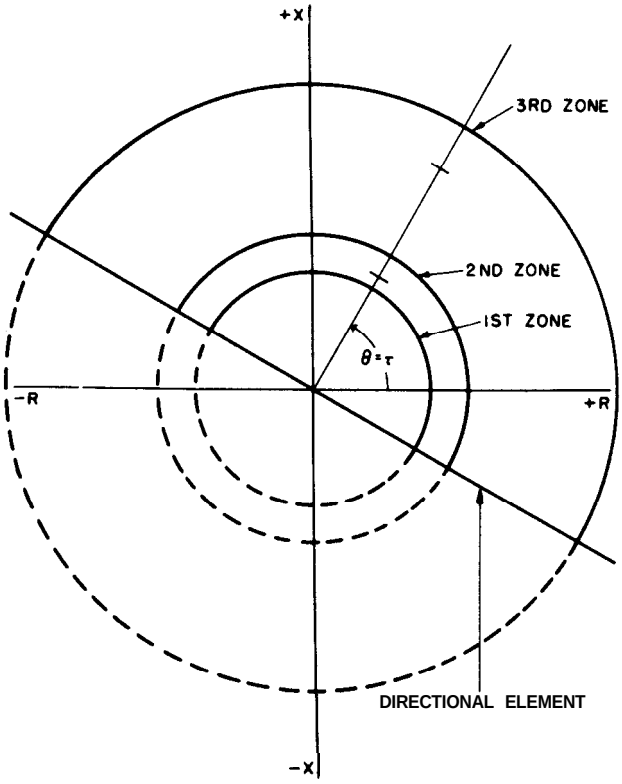


Fig. 8. Three-step Impedance Relay Characteristic
(For Line of Fig. 7)

The contacts of the starting mho element and the reactance element are in series so that relay tripping is confined to those areas within the boundaries shown by the solid lines of Fig. 9. This prevents relay operation for faults in the reverse direction or on load currents.

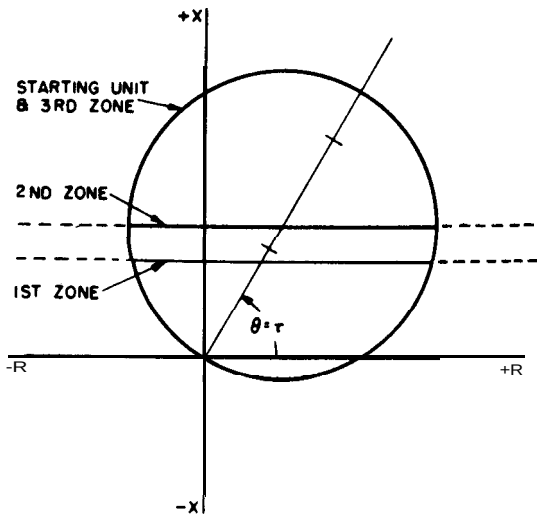


Fig. 9. Three-step Reactance Type Relay Characteristic
(For Line of Fig. 7)

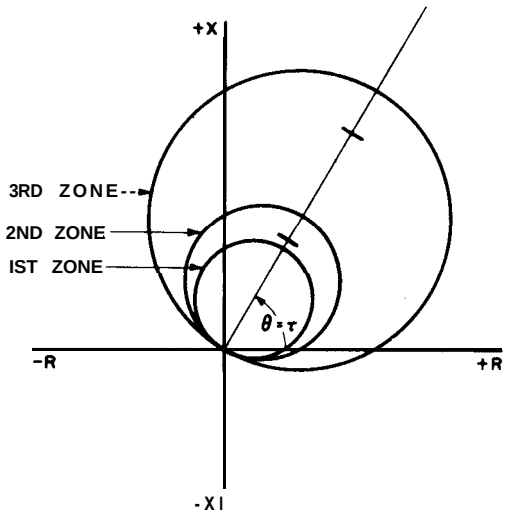


Fig. 10a. Three-step MHO Relay Chorocteristic
(For Line of Fig. 7)
No Offset

(3) The Mho Relay is composed of three mho elements, each adjusted for a different reach. It will be seen that there is no need for a "limiting" element such as required in the other two relays, since the mho element closely approaches the ideal relay element. In the practical relay, the 3rd zone element may be used as shown in Fig. 10a, or with reversed 3rd zone as shown in Fig. 10b. In either case, the 3rd zone may be offset to some extent, depending upon the application requirements.

Out-of-step blocking relays, in general, utilize one of two basic elements: (1) an impedance element; (2) the mho element. Examples of these

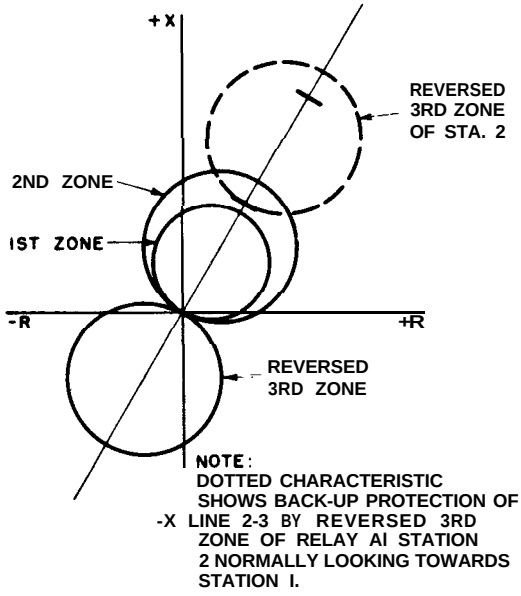


Fig. 10b. Three-step MHO Relay Chorocteristic
(For Line of Fig. 7)
Showing Reversed 3rd Zone

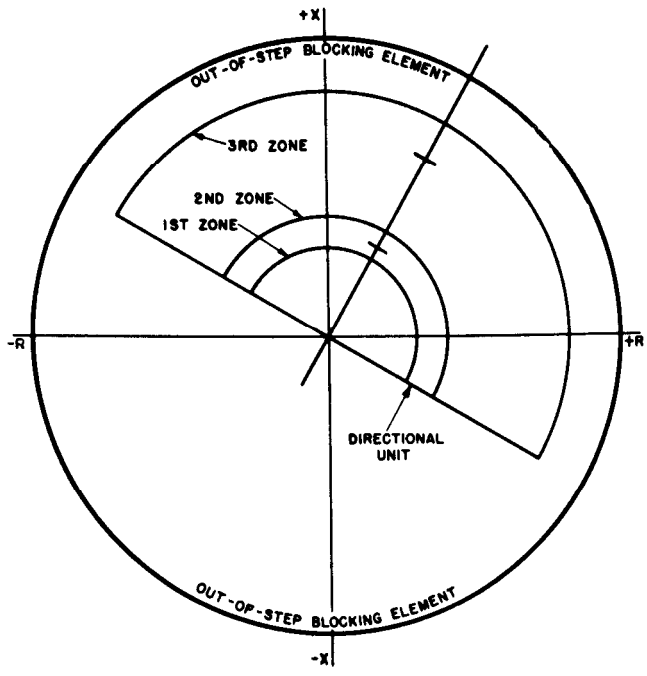


Fig. 11a. Out-of-step Blocking of Impedance **Relay Using an Impedance Element**

relay characteristics are shown in Fig. 11a and llb. Further discussion of this function will be given later.

In the foregoing discussion, there were a number of instances wherein interesting and pertinent

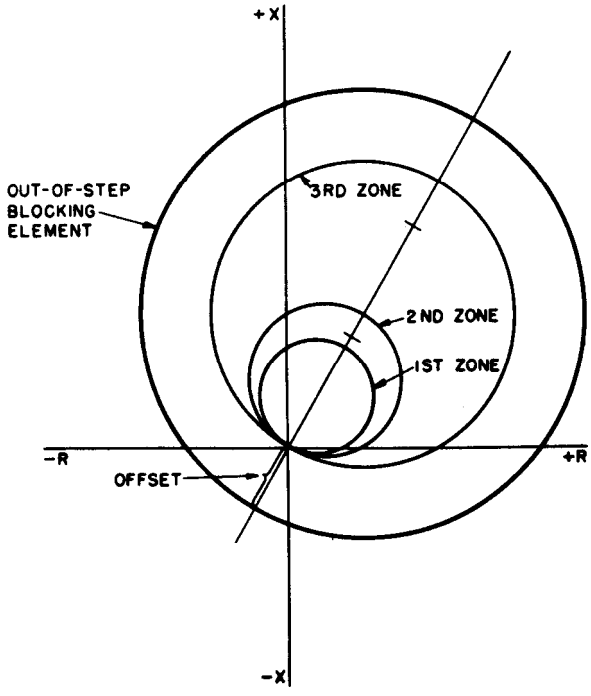
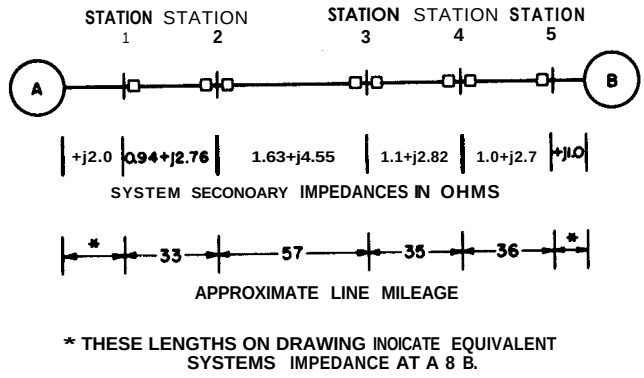


Fig. 11b. Out-of-step Blocking of MHO Relay Using *an Offset MHO Element*



* THESE LENGTHS ON DRAWING INOICATE EQUIVALENT SYSTEMS IMPEDANCE AT A 8 B.

Fig. 12. One Line Diagram of System Used in *Examples*
(Single Circuit Line)

questions were avoided; foremost among these would be the question, "Why show these characteristics on the R-X diagram?" We have seen that it is convenient to do so from the viewpoint of plotting their operation in simple curves, but that is scant justification until we consider how various system conditions appear on the same plot. This will then be the next step in our discussion.

SYSTEM CONDITIONS ON THE R-X DIAGRAM

In order to use the diagram, we must begin with an equivalent two-machine system. The impedances used may thus represent combined systems on either end of the line we are studying. In our case we will assume a system reduced to the case of Fig. 12. Here the system ohms have been changed to "secondary ohms." We can now plot the system impedance on a "secondary-ohms" impedance diagram, as shown in Fig. 13, by the following steps: Starting at Gen. A, the secondary impedance to an imaginary fault at Station 1 is (0 + j 2.0) ohms. The + j 2.0 is plotted along the X axis in the positive direction (Fig. 13a). If we move our imaginary fault to Station 2, the secondary impedance is now (0 + j 2.0) + (0.94 + j 2.76) = (0.94 + j 4.76) ohms (Fig. 13b). We can continue in this manner until we reach the end of the system, i.e., back of the reactance of the equivalent generator at B (Fig. 13c). Our equivalent system impedance total is then (4.67 + j 15.23) ohms. The resulting R-X diagram of the system, the "system impedance line" and the "system impedance center"* are shown in Fig. 13d.

We should pause here briefly to review and emphasize some of the conventions we have automatically adopted.

1st. The units of R and X are to the same scale, e.g., one ohm resistance equals one horizontal scale unit, one ohm reactance equals one vertical scale unit.

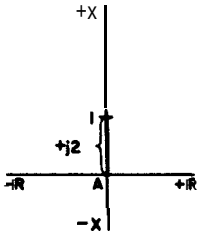


Fig. 13a. Fault at Station I as Seen from A

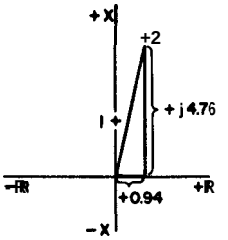


Fig. 13b. Fault at Station 2 as Seen from A

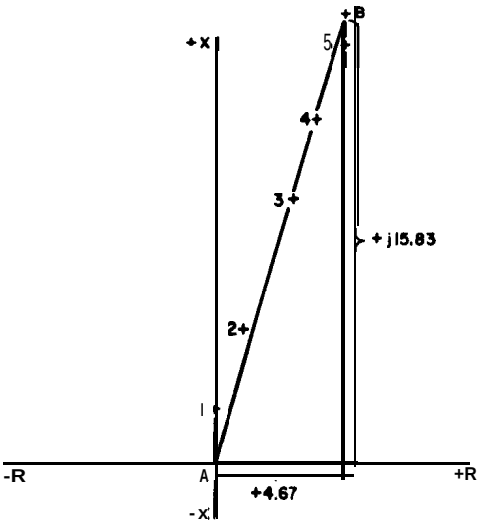


Fig. 13c. Fault at Station B as Seen from A (Other Stations as located Previously)

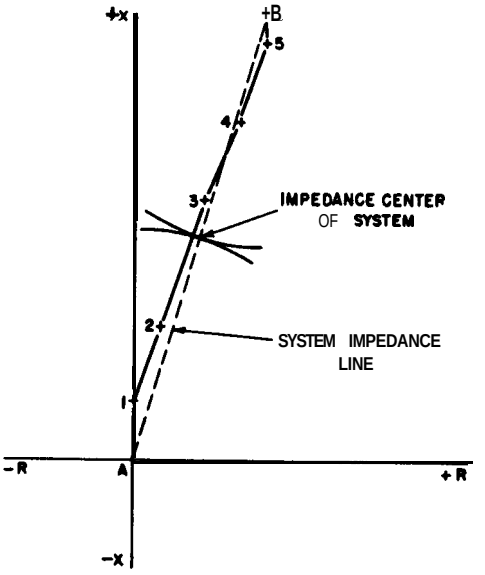


Fig. 13d. R-X Diagram of System Showing System Impedance line and Impedance Center of System

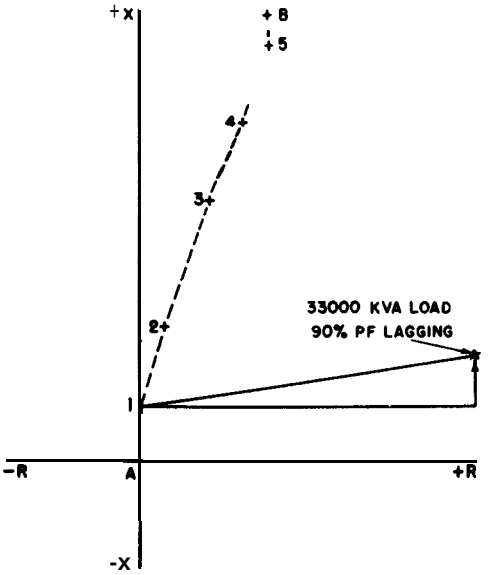


Fig. 13e. large load of Station 1 Single End Feed from Station A

Fig. 13. Construction of the R-X Diagram for the System of Fig. 72

2nd. We have adopted a sign convention of plus R to the right and plus X to the top, as seen looking towards Station B. The importance of this sign convention will be realized when we temporarily place a large load of lagging power factor, say at Station 1 with Station B disconnected. In this case we would see a large +R and a smaller +X from Station A (Refer to Fig. 13e). (The value plotted represents a load of approximately 33,000 kva, just twice normal circuit loading for this line.)

3rd. The sign conventions hold only when we "look" in the same direction as we did when we plotted the diagram. That is, a fault at Station 4 (or a lagging power-factor load at Station 4) when viewed from Station B, obviously must appear as +R and +X. Strictly speaking, a new R-X diagram should be plotted for this condition, but the mental gymnastics involved are of little difficulty when compared with the advantages of using the same diagram for both directions in this simple case.

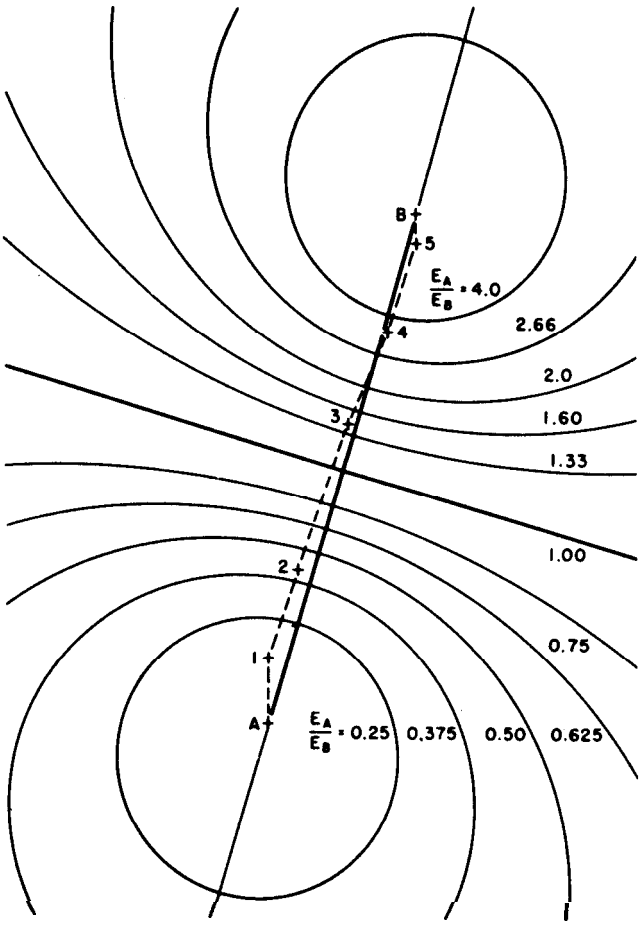


Fig. 14. Constant Voltage Ratio Circles for System of Fig. 12

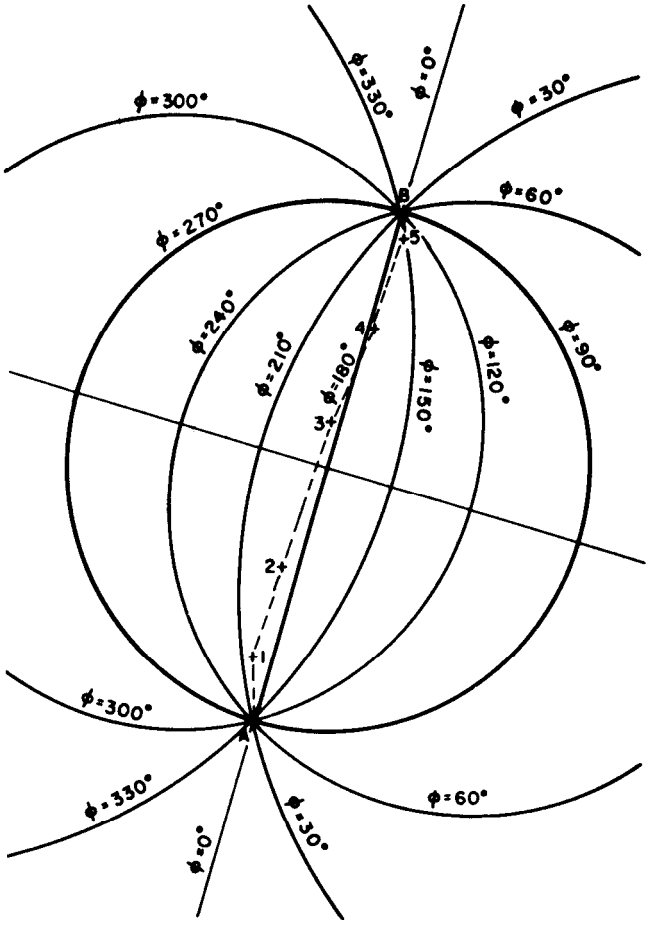


Fig. 15. Constant Angular Separation Characteristics for System of Fig. 12

4th. The location of the origin was assumed to be back of the generator reactance at A. This location is convenient for load transfers and power swings between stations when viewed from A. It is not so convenient when considering relay operations at the intermediate stations. Fortunately, we can avoid the inconvenience of subtracting the vector impedance between Station A and the intermediate station by locating the origin at the relay location. This will enable us to measure directly the impedance seen by that relay for various system conditions.

We are now ready to investigate various system phenomena, as interpreted by the R-X diagram,

One of the earliest attempts at determining qualitative relay performance data during swing conditions resulted in the development of a new concept of system and relay performance. C. R. Mason presented an AIEE paper[1] in 1937 in which he analyzed relay performance during swing conditions. The results of his analysis were summarized in plots of relay torque as a function of the separation angle between the two machines of his equivalent system. At the same time Mr. J. H. Neher presented the first paper[2] on distance-relay characteristics plotted on an impedance diagram, and later gave Mason valuable suggestions in a discussion of Mason's paper[1]. Neher pointed out that, for equal voltages back of machine reactances (EA/EB = 1.0), the apparent system impedance varied (as the two machines slipped a pole with respect to each other) along a line which was the perpendicular bisector of the system impedance line, In the closing discussion of his paper[1], Mason then showed this "swing line" on the same R-X diagram with different relay characteristics, This 'was indeed a true pioneering step, for it offered a readily understandable analysis of a problem that was becoming increasingly acute.

Subsequent authors[3,4] have enlarged and advanced this concept beyond the limitations of this

[1,2,3,4] For numbered references, see list at end of paper.

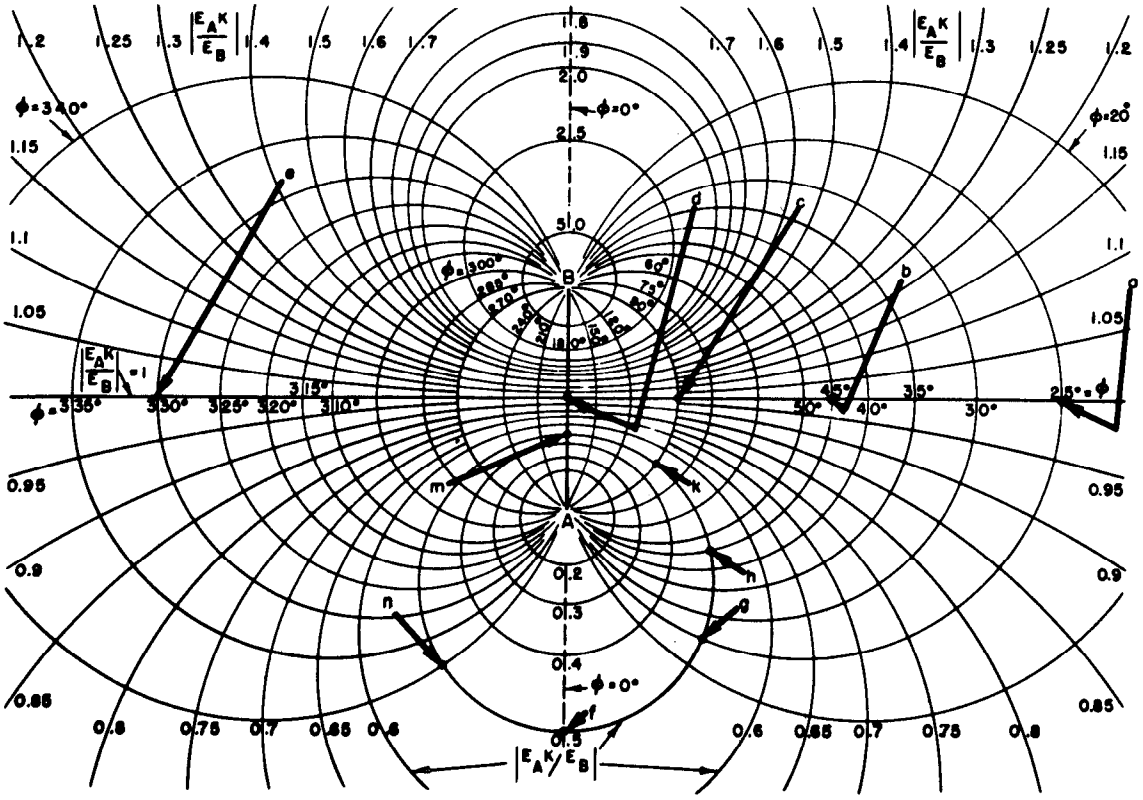


Fig. 16. General Per-unit Impedance Diagram

particular case and developed curves for various ratios of EA/EB. In this work it was proved that the apparent impedance during out-of-step conditions followed a definite circle for each value of EA/EB. These circles were all centered on the system impedance line with radii and off sets determined by the various values of this voltage ratio. These characteristic circles are shown on Fig. 14. Note that the specific case of EA/EB = 1.0 is but a logical limit to the increasing circular characteristics (the radius and offset for this case are each infinite). NOTE : Certain simplifying assumptions have been made for this discussion. The most important of these are: (1) the system can be represented by a single circuit between Station A and Station B ; (2) the effective excitation voltage remains constant; (3) the machine impedances remain constant.

The mathematics and curves for the generalized case have been summarized and presented in a paper by Miss Edith Clarke[4]. In this paper Miss Clarke presents another series of curves well worth our attention. This series concerns those circles of equal angular separation. If the angular separation of the two machines is held constant while the voltage ratio is varied, the apparent impedance will trace a portion of a circle which passes through both A and B and whose center lies on the perpendicular bisector of the system impedance line. The radii and offsets of these circles are determined by the angular separation between A and B. These circles are shown on Fig. 15. These circles have a peculiarity that should be noted. The line A-B is a portion of the circle (having infinite radius) which represents 0 and 180 degrees separation, and it also separates the right and left portions of each constant angular separation circle into two characteristics, wherein the separation angle for the two parts differs by 180 degrees. Thus, the 90-degree separation circle on the right becomes the 270-degree separation circle on the left.

The foregoing comments will become clearer if we consider the general per-unit diagram[4], Fig. 16, the use of which greatly reduces the time of calculation, as illustrated in a discussion of Miss Clarke's paper[5]. In using this diagram, it is necessary to shift the vertical axis to coincide with the impedance angle of the system. A detailed discussion of this diagram and its use is not in order; however, it will be helpful to trace two cases on the diagram. In the first instance, assume EA/EB = 1.0. Thus, our swing characteristic is the horizontal line. Suppose we start our swing when A leads B by 25 degrees, we would then trace a path: a-

[4]For numbered references, see list at end of paper.

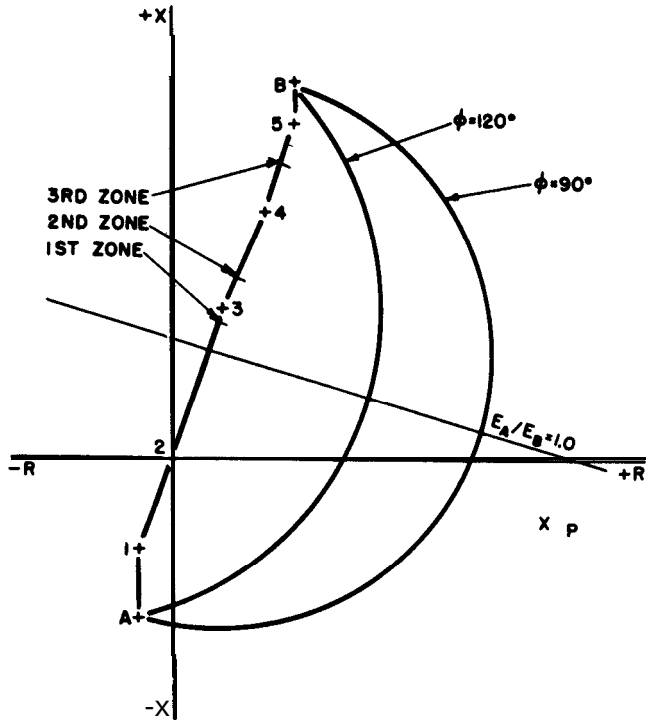


Fig. 17. Working Example for System of Fig. 12

Showing: $\frac{E_A}{E_B}$ = 1 .O Swing Line
ϕ = 90°, 120° Separation Curves
Point P for a 27,000 Kw Load Flow
Relay Zone Reaches

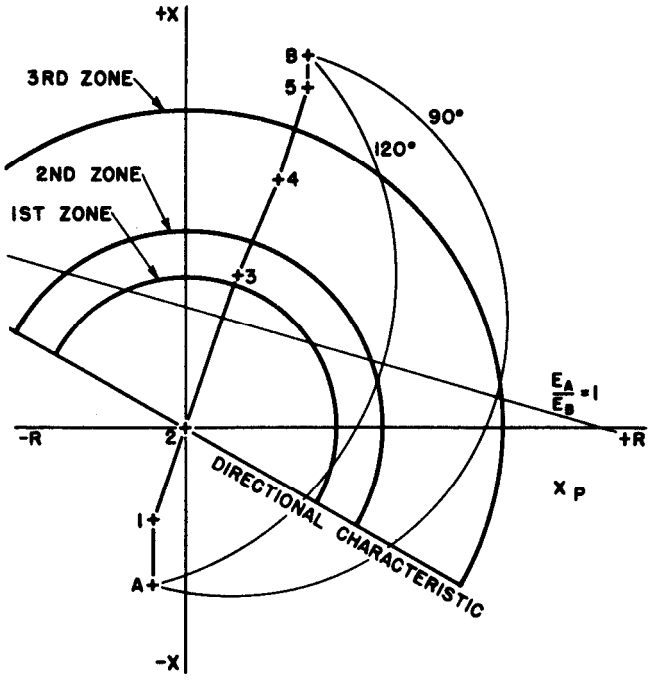


Fig. 18. Impedance Relay Superimposed on Working Example (Fig. 17)

25°, b—45°, c—90°, d—180°, e-330". It will be helpful to trace another path, say for EA/EB = 0.5. At zero degrees separation, machine A "sees" what is predominantly -X (point f of Fig. 16) or capacitive reactance. This will jibe with operating experience when we consider that this appears as a highly inductive load to machine B. Once again n-e can trace points around this circle for various angles : g—25°, h—45°, k—90°, m—180°, n—340". Before we dismiss this diagram, we should note the 90-degree separation circle. This circle is centered at the system impedance center and passes through both A and B. For any value of EA/EB, the apparent impedance of the system must pass through this circle as the separation angle reaches and passes 90 degrees. This circle, for 90-degree separation, is an important one for us to remember for two reasons : (1) It is easy to construct, and (2) it indicates an approximate limit, in angular separation, for power swings of the system if steady-state stability limits are not to be exceeded.

This limited discussion has indicated that the R-X diagram is a useful tool in describing and analyzing various system conditions. We have seen, in the previous discussion, that it is useful in studying relay characteristics. Our next step is thus the combination of the relay characteristic and system characteristic on one diagram.

COMBINED SYSTEM AND RELAY CHARACTERISTICS ON AN R-X DIAGRAM

Refer to Fig. 17. This is the R-X diagram of our system of Fig. 12 with several references superimposed thereon. The EA/EB = 1.0 power swing line is shown, as are the 90°— and 120°— separation circular arcs. In addition, we now have a point P indicating the impedance seen for an interchange loading of aproximately 27,000 kw. Note that the origin has been shifted to Station 2 of Fig. 12. We will consider various distance relays for the line between Stations 2 and 3, each having lst, 2nd, 3rd zone reaches as indicated on our working example, Fig. 17.

The impedance relay characteristic for this case is shown, superimposed on the working example, in Fig. 18. The directional unit is necessary to prevent relay tripping for faults between Station 2 and Gen. A.

Notice that third zone tripping will occur for any separation beyond 90 deg (EA/EB = 1.0) and for even less angular separation than 90 deg if EA/EB < 1.0. Furthermore, at certain values of EA/EB, the 1st zone instantaneous trip area extends into the 120-degree separation area. Since 120" separation is an approximate transient stability limit for most systems of this pattern, some form of blocking for power swings is almost man-

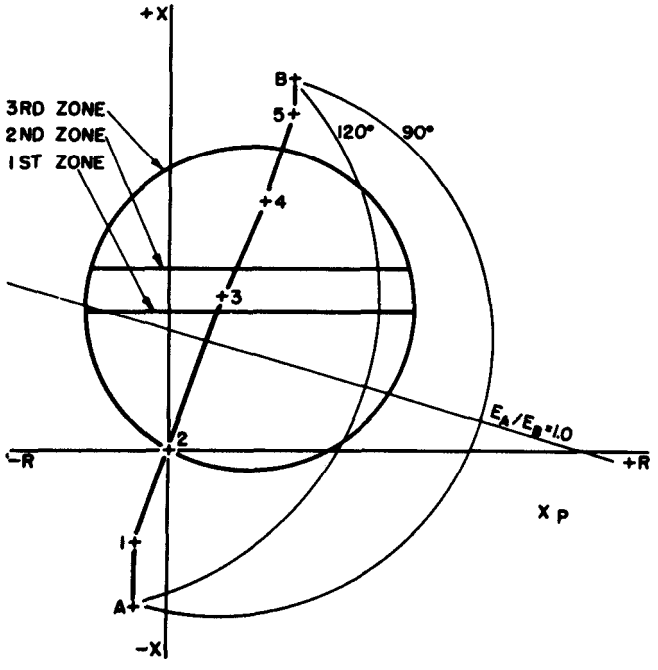


Fig. 19. Reactance Relay (Type GCX) Superimposed on Working Example (Fig. 17)

datory. If we attempt to provide out-of-step blocking for the complete relay, we find that our blocking impedance element would come into the load area (see Fig. 11a) . This, of course, is intolerable if we expect to carry appreciable load over this line since our line relays would be continuously blocked during heavy load transfer periods.

The Reactance Relay with mho starting element is shown superimposed on the working example in Fig. 19. Again, a glance will show the need for the limitation the starting unit characteristic imposes upon the reactance element. For example, load flow towards B, for all practical voltage ratios (EA/EB), will fall in the operating range of the reactance element. The tripping range of the relay is limited by the starting unit characteristic so that the relay tripping area is far removed from even the heaviest load areas. It is interesting to note that this 3rd zone unit, having the same distance reach for faults as the impedance unit (see Fig. 18), nevertheless lies well within 90-deg separation curve. Further comparison of Fig. 18 and Fig. 19 will show, however, that the reactance relay for this line is more vulnerable than the impedance relay to instantaneous operations on severe swings that extend to the 120-deg region.

Let's pause here a moment and review the causes of our difficulties in applying a relay to the line used in this example. One source of these difficulties is the fact that this line spans the impedance center of the system. Thus, the apparent impedance of any severe power swing will move in and out of the operating area of the relay characteristic, reaching points well within and near the center of this area. It is therefore essential that the dimension of the relay characteristic, in the direction of the approaching swing characteristic, be as small as possible. This is particularly necessary in the case of the instantaneous and high-speed elements. A second source of difficulties, which serves to compound the effects noted above, is that this is a long line ; long as determined by the "secondary ohms" of the line (4.8 ohms in this case) and long in the sense that it comprises an appreciable percentage of the total system impedance. The two aspects of this statement will become clearer if we consider that the first serves to determine the necessary size of the relay characteristics on the R-X diagram whereas the second serves to determine the relative size of the relay characteristic to the system characteristics ; e.g., the size of the 90" or 120" separation curves.

These considerations lead us to an investigation of the mho relay for this application. A composite picture of the reactance relay characteristic of Fig. 19 and of the mho relay characteristic with the same "reach" for each corresponding zone is presented in Fig. 20. In order to emphasize the value of the mho characteristic in this case, the

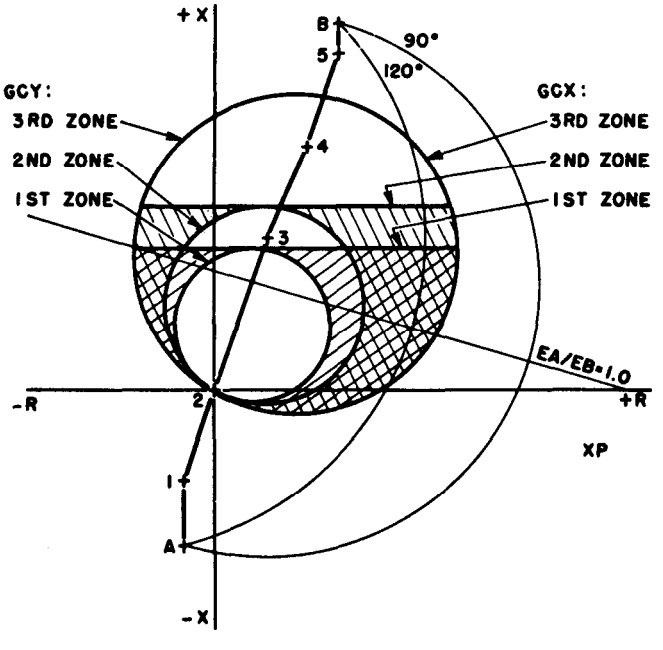


Fig. 20. MHO Relay (Type GCY) Superimposed on Working Example (Fig. 17) Showing Comparison to Reactance Relay (Type GCX)
e. g. Second Zone Trip Area Saved Shown
First Zone Trip Area Saved Shown

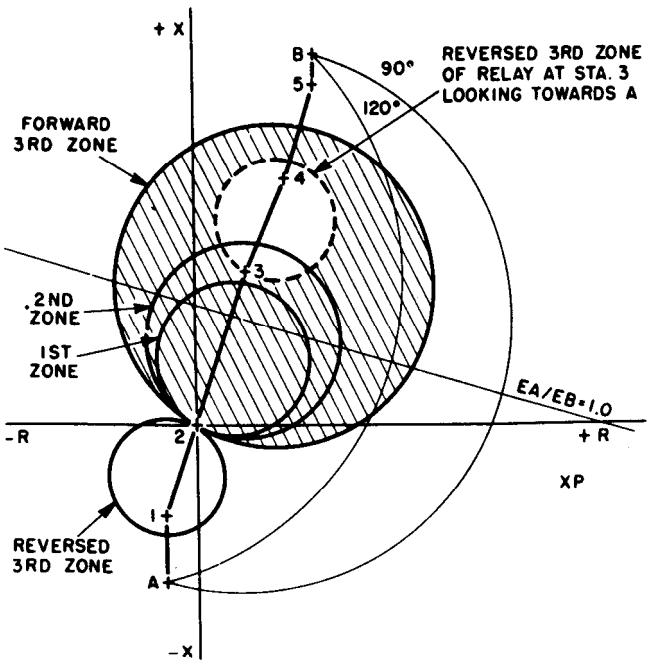


Fig. 27. MHO Relay Type (GCY) Superimposed on Working Example (Fig. 17) Showing Forward and Reversed Third Zone Comparison
(Excess Tripping Area of Forward Third Zone Shown Shaded)

amount of tripping area "saved" in the first and second zones has been shaded. That is, each shaded area represents those regions wherein the reactance element would trip and the mho element would not trip in the event the apparent impedance of a power swing entered that area. Fig. 20 illustrates that the mho relay, in this case, is not susceptible to either instantaneous or high-speed tripping during power swings except for those swings that exceed 120-degree separation.

We have materially reduced the 1st and 2nd zone area by the use of the mho relay but have not changed the area covered by the 3rd zone. This can be accomplished through the use of the reversed 3rd zone. The GCY relay characteristic was shown with this reversed 3rd zone in Fig. 10b. If we apply this treatment to the line under consideration, we would assign the 3rd zone of the relay at Station 2 to back-up protection for the line from Station 2 to Station 1. The responsibility for providing backup protection for faults in the line section between Station 3 and Station 4, formerly assigned to the relay at Station 2 (looking towards Station B), will now be assigned to the relay at Station 3 (looking towards Station A) by reversing its 3rd zone. The resultant sawing in tripping area of the 3rd zone element providing this back-up protection is shown by the shaded area on Fig. 21. This illustration shows the great reduction in the lateral dimension of the relay tripping area which is possible through the use of the mho-type relay with the reversed 3rd zone. In this case, we have succeeded in keeping the tripping characteristic of the entire relay well out of those areas which have been used to delineate violent power swings for a stable system condition.

OUT-OF-STEP BLOCKING

We have seen that the apparent impedance follows a definite curve during swing and out-of-step conditions, the particular curve being dependent on the voltage ratios. It is apparent that (since this curve crosses the line between A and B at 180°), when the system is near 180° separation angle, the apparent impedance can be the same as the impedance to a fault in the line. How, then, can a relay differentiate between the two?

Consider Fig. 22. If the system is carrying the interchange load shown (Point P), a fault on line section 2-3 results in a change of impedance from P to F in practically zero time. On the other hand, during the first few swing cycles for the out-of-step condition, the apparent impedance drifts through point M (EA/EB = 1.0) at relatively slow speed. Therefore, we will set up the offset mho blocking element (shown by the heavy line on Fig. 22) to block the 1st and 2nd zones of the mho relay

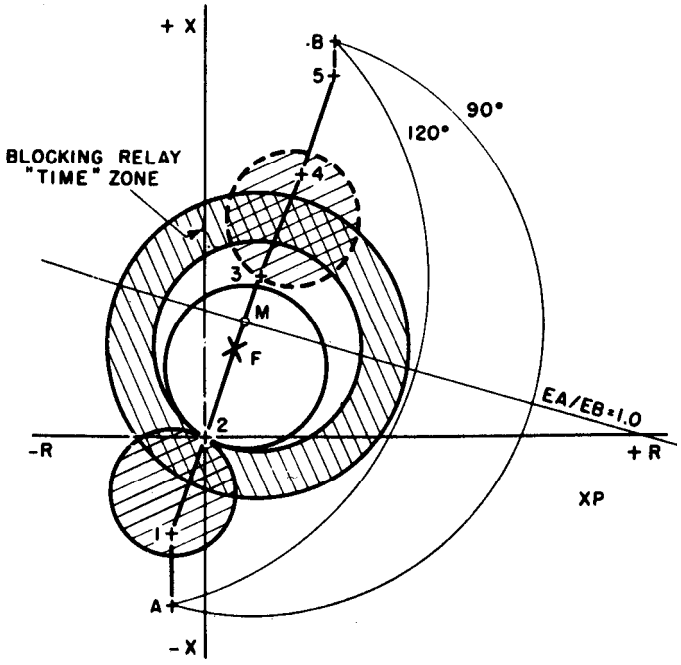


Fig. 22. MHO Relay (Type GCY) with Reversed Third Zone Superimposed on Working Example (Fig. 17) Showing Offset MHO *Out-of-step* Blocking Relay
1st & 2nd Zones Blocked Only
3rd Zones Not Blocked

if the time required for the impedance to change from any point outside the blocking characteristic to any point within the shaded area exceeds a predetermined minimum time. In other words, blocking will be realized if the blocking element picks up before the relay tripping element picks up, with sufficient time interval between the two so that certain auxiliary relays will have time to operate. Blocking will not be realized if both blocking and tripping elements pick up simultaneously.

The blocking element characteristic must therefore surround the largest tripping element characteristic which it must block with sufficient margin to allow blocking for the fastest swings anticipated. It must meet this condition and yet be as small as practical to avoid unnecessary operation. Therefore the choice of blocking elements is affected by the considerations that influence the choice of tripping elements for the line under consideration.

OUT-OF-STEP TRIPPING

Recognizing that systems can and do occasionally go out of step in spite of the best efforts of system designers and system planners, we are faced with the problem of minimizing the effects of this out-of-step condition for the greatest possible portion of the system load. It is certainly desirable to avoid outages, yet the system must be split in order to permit a correction of the initial cause of instability and to enable us to reload the system. The best way to split the system, since we must, is to divide it so that each area thus isolated has sufficient generation to carry its load insofar as this is possible. This calls for definite, preselected tripping points. If this function were left to the line relays, as we now know them, the system split point would be determined by several factors which might vary from hour to hour. (E.g., system var scheduling, which could change E_A/E_B; system loading, which could change the total system impedance and/or the proportions of the system impedances about a given relay.) These changes assume greatest importance when the "normal" impedance center passes through or close to an important bus. Thus, it is necessary to use a reliable protective relay that will trip for those out-of-step conditions which affect the circuits it protects and will not trip for other normal or abnormal system conditions.

Let us examine these out-of-step characteristics in order to determine the requirements for an out-of-step tripping relay. The basic distinction of an out-of-step condition can be shown by the fact that the apparent impedance, as seen from any one point in the system, changes from:

1. a point to the right of the system impedance line,
2. to a point on the system impedance line,
3. to a point to the left of this line.

This sequence applies to that case wherein Machine A advances ahead of Machine B, and the impedance is "seen" looking towards Machine B. If either of these latter conditions is reversed, the sequence of impedance changes will reverse. If both conditions are reversed, the apparent impedance will follow the same sequence described above.

Another characteristic of an out-of-step condition is that the impedance change occurs over a finite period of time which (for the first few slip cycles, at least) is long compared to the impedance change directly associated with a fault.

Our task now is to design an ohmic relay which will recognize these distinctive characteristics. One straightforward solution would be to use two ohmic characteristics to divide the R-X diagram into three areas, and use auxiliary relays to time and analyze the impedance swing through these areas. Returning to Fig. 15 briefly, we see that two mho elements, properly offset and adjusted, could give us circles which, for example, could be equivalent to the 165°-345° circle for one and the 195°-15° circle for the other. The useful part of these circles would lie between Points A and B of our R-X diagram (i.e., the 165" arc and the 195" arc). However, for practical values of E_A/E_B, these arcs can be closely approximated by straight lines. Thus, we can use two "reactance" elements, each having an angle of maximum torque perpendicular to the system impedance line. That is, the operating characteristics would be straight lines and the elements would be capable of adjustment so that these lines would be parallel to the system impedance line. The "pick-up" setting would then determine the distance between these characteristics and the system impedance line, i.e., the amount of "offset."

We have drawn the characteristics of two such elements on our working example, as shown in Fig. 23. In order to determine the practical operating requirements, we have used exaggerated settings in this illustration. We will design each of these elements with two contacts which we will designate as No. 1 and No. 2. The No. 1 contact will be closed whenever the apparent impedance falls to the left of the element characteristic, while the No. 2 contact will be closed whenever the apparent impedance falls to the right of the element characteristic. For discussion purposes, we will call the left element "A", and the right element "B", and shade those areas where each No. 1 contact is closed. This has been done on Fig. 23 to show three definite areas :

Area No. l-contacts A_2 and B_2 closed,
Area No. 2—contacts A_2 and B_1 closed,
Area No. 3—contacts A_1 and B_1 closed.

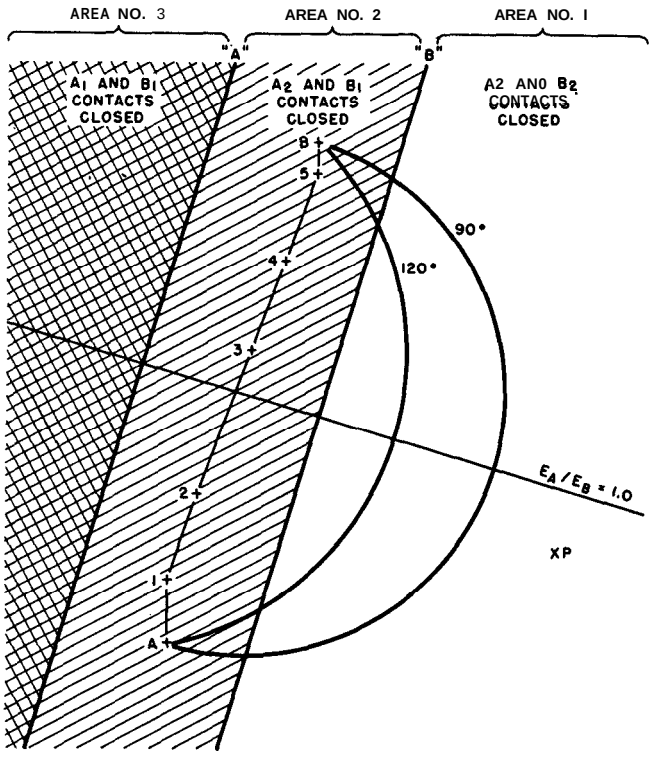


Fig. 23. Out-of-step Relay (exaggerated setting) Superimposed on Working Example, Fig. 77

Before we proceed to investigate the requirements of the auxiliary relays, we should pause to remember that the extensions of our system impedance line beyond Points A and B intersect the "curves" for zero degrees separation between Machines A and B. This brings to the fore the case of a machine "floating" on the line, i.e., carrying little or no power. Turbine governor and system characteristics are such that the power flow from such a machine is apt to be quite erratic in direction, although small in magnitude. Nevertheless, these "swings" may cause operation of the "reactance" elements. "Out-of -step tripping" would be embarrassing in such a case, so we will use an overcurrent relay to "supervise" the auxiliary relay chains. This overcurrent relay simply insures that tripping will occur only when the "swing" currents are at least of the same order of magnitude as load currents.

The auxiliary-relay chain, with the overcurrent relay supervision, is shown in Fig. 24. Each of the auxiliary relays has a pick-up time of 0.005 second and a drop-out time of approximately 0.1 second, with the exception of X_3 and X_6 which have adjustable long-time-delay drop-out. We are now ready to check the over-all relay design.

Let us assume an out-of-step swing occurs which starts from some loading point out on the right on the $E_A/E_B = 1.0$ swing line and progresses with Machine A leading B. A, and B_2 are closed initially, X_1 will pick-up through B_2 as soon as the overcurrent relay closes its contacts. When the swing progresses across the "B" element characteristic, B_2 opens and B_1 closes. Since X_1 has a slight time delay drop-out, X_2 will be energized and remain energized through its own contact as soon as B_1 closes. As the swing progresses across the "A" element characteristic, A_2 opens and A_1 closes. Contact A, energizes X_4 which causes X_3 to be energized and seal in through its own contact around that of X_2. X_3 will remain energized for the remainder of the slip cycle until the overcurrent relay drops out or the "A" characteristic is crossed again. If X_3 is used for tripping, the relay will have performed its function; however, for other applications it is desirable that X_3 be a relay with adjustable long time drop-out.

A similar examination will show that X_6 will be energized for a slip cycle in the reverse direction. In both cases, it is necessary that the characteristics of both elements be crossed before X_3 or X_6 can be energized. Thus, the occurrence and subsequent clearing of any fault in the system cannot cause relay operation unless the system is shaken so that the machines continue their separation beyond 180°. (Both X_3 and X_6 have drop-out times

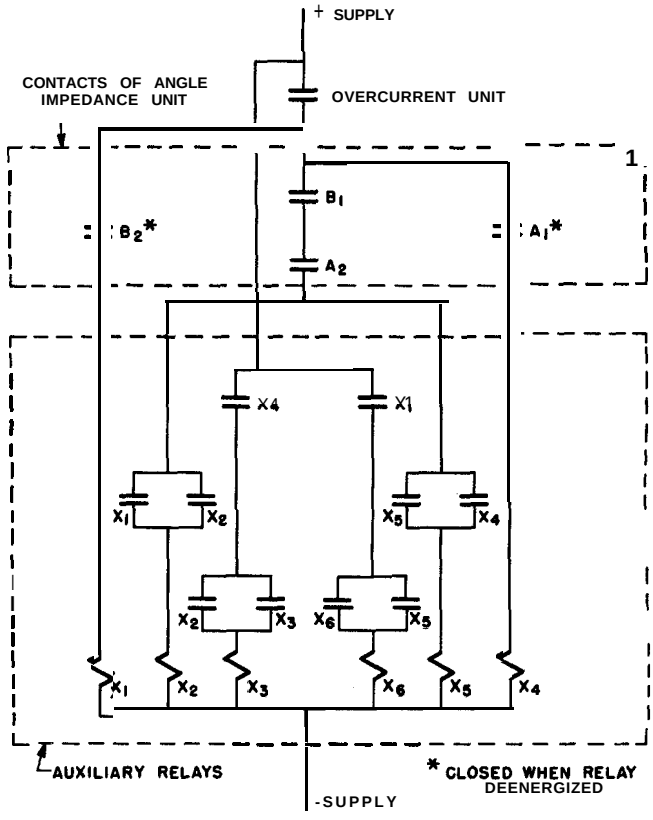


Fig. 24. Auxiliary Relay Connections for Out-of-step Protective Relay

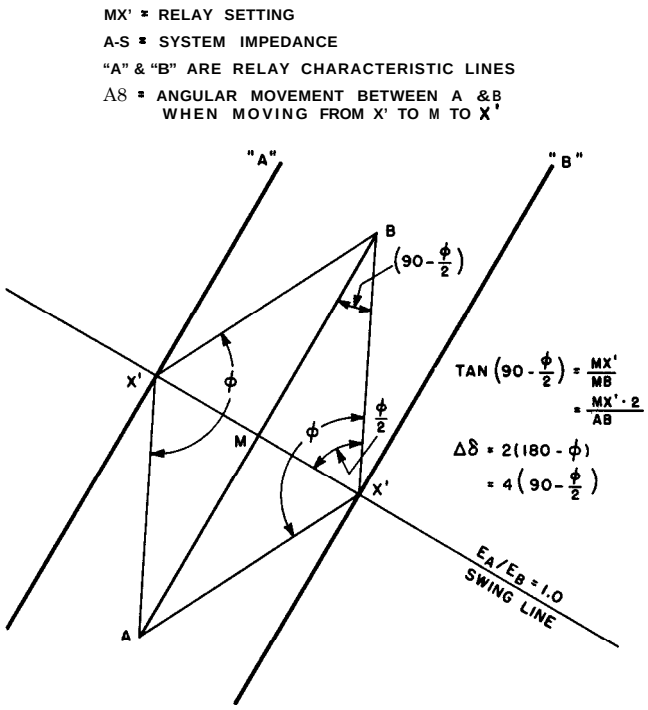


Fig. **25a.** **Geometric Relationship of Out-of-step Relay to System Characteristics**

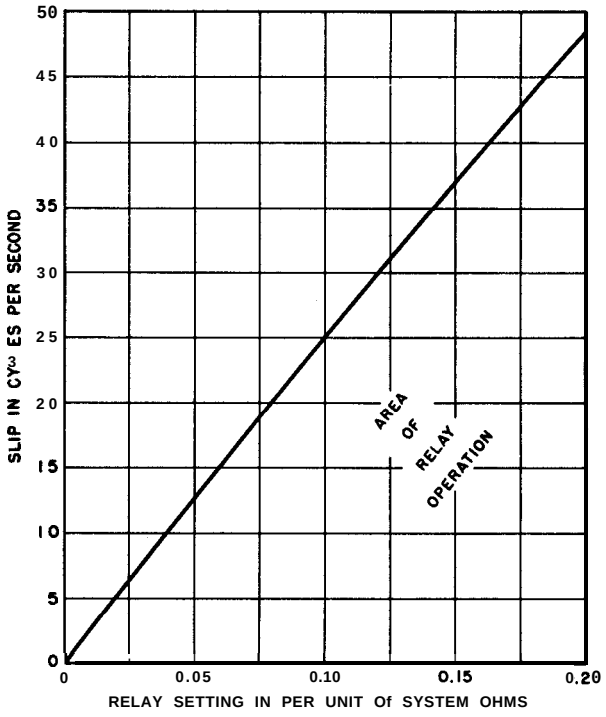


Fig. 25b. Minimum Allowable Relay Setting for Operation of Out-of-step Relay Versus Maximum Expected Slip

adjustable between 0.5 and 3.0 seconds. This assures that X_3 will remain "picked-up" for all except the longest slip cycles and is useful where the "slip-directional" feature of the complete relay is used to change prime mover input.)

When Fig. 23 was introduced, the ohmic settings of these "reactance" elements were said to be exaggerated. Let us investigate the requirements for these settings and plot the resultant settings on our working example.

The first requirement is that the relay operate for the fastest slip cycle expected on the first swing. The most critical operation then, is that of closing the contacts of X_2 during the period when the swing is traversing the area between the "A" and "B" characteristics. Since X_2 has a time-delay of 0.005 second in closing its contacts, the swing must remain in this area at least 0.005 second. The angle through which the system moves in traversing the area between the relay characteristics we will call AS. Since the swing must take at least 0.005 second to pass through this angle, the maximum slip which will permit relay operation is given by the equation:

$$S_{max} = \frac{1}{360} \times \frac{\Delta\delta}{0.005}, \text{ or;}$$

$$S_{max} = \frac{\Delta\delta}{1.8} \text{ slip cycles per second.}$$

It remains to express AS in terms of the relay setting.

If we assume that the relay is located at a station which lies on the system impedance line and that the relay characteristics parallel the system impedance line, we can show the relationships existing between the relay setting, the system impedance, and the separation angle by the diagram shown in Fig. 25a. The separation angle, Φ, is the angle of separation between Machines A and B as determined by the intersection of the relay characteristic with the "equal-excitation-swing-line." In Fig. 25a, AB is the system impedance, MX' is the relay setting (equal offset for each characteristic) and M is the mid-point of AB. In traveling from "B" to "A", we must move from a system angle of Φ to 180° (Point M) to an equal angle on the other side of 180°. Our total angular travel, AS, is thus equal to 2 x (180 $-$ Φ), or:

$$\Delta\delta = 4 \times (90 - \frac{\Phi}{2}).$$

From the geometry of Fig. 25a, we see that the :

$$\tan (90 - \frac{\Phi}{2}) = \frac{MX'}{MB} = \frac{MX' \times 2}{AB} \text{ or:}$$

$$(90 - \frac{\Phi}{2}) = \tan^{-1} \frac{(2 \times \text{relay setting})}{\text{system ohms}}$$

Substituting these values in the previous equations, we can write :

$$S_{max} = \frac{4}{1.8} \tan^{-1}\left(\frac{2 \times \text{relay setting}}{\text{system ohms}}\right)$$

From this last equation, we can plot a curve showing the relationship between the relay setting, expressed in per unit of system ohms, and the maximum expected slip which can give successful relay operation. This is shown in Fig. 25b.

Reference to Fig. 15 will show that the relay characteristics cover the least spread of separation angles at their intersection with the "equal-excitation-swing line." This means that, all other things being equal, at constant slip speed the apparent impedance will cross the relay characteristics faster for the condition of $E_A/E_B = 1.0$ than for any other excitation ratio.

The curve of Fig. 25b may be used with only slight theoretical error for those cases wherein the relay "station" does not lie on the system impedance line, even for values of "per unit" relay settings beyond 0.20. This is true because of the relative linearity of the tangent function for small angles (less than 20 degrees).

The question naturally arises: "What would be the maximum slip to be expected on the first swing cycle?" Any answer to this question would involve various system parameters and should be based on a transient stability study of the system under consideration. Since we do not have such a study available for the system used in our examples, we will assume an answer based on experiences with other systems and on a rough approximation.

Investigation of several transient stability studies made on the Network Analyzer in Schenectady shows an average rotor velocity of 1.5 slip cycles per second. This is in the region of 180° separation between that machine with greatest velocity and the rest of the system. This gives us one bench mark.

Another bench mark may be found by a rough approximation. One of the most severe cases in transient stability studies will be found in the condition of a direct three-phase fault on the terminals of a machine which had been fully loaded up to the instant of the fault. In this case, all of the prime mover torque is available for rotor acceleration except a small portion appearing as I^2R loss in the armature windings. (This statement is true within the usual limits of a transient stability study. See reference 10 at rear of book.) As an example in this case, we will investigate a 60 megawatt preferred standard machine, fully loaded at the 119 percent rating of the turbine, rated power factor and rated terminal voltage. Average per unit constants for this machine are:

$X'd = 0.13 \qquad r_a = 0.00111 \qquad H = 3.8$

(See reference 10 for definition of terms.)
Machine loading is 0.935 + j0.58
$V' = 1.0 + (0.935 - j0.58)(j0.13) \approx 1.083$

$$I' = \frac{1.083}{0.13} = 8.33$$

$(I')^2 R_a = 0.077$

Turbine output = 0.935

Acceleration torque = 0.935 − 0.077 = 0.858

$$\text{Acceleration constant} = \frac{180 \times 60}{3.8} = 2840°/\text{Sec}^2$$

$$\text{Acceleration} = 0.858 \times 2840 = 2440°/\text{Sec}^2$$

Under this constant acceleration, the time required for the machine to advance 180" would be 0.384 second and the velocity at that time would be 938" per second or 2.6 slip cycles per second. These conditions are not directly applicable to our problem ; however, they do serve to furnish a bench mark in selecting our relay setting.

The second requirement for the relay setting is that the relay characteristics should "blanket" the system plot on the R-X diagram. That is, a fault any place in the system should fall between the relay characteristics. As an illustration, let us refer back to Fig. 23 and consider the application of this relay at Station 2. Since the relay characteristic will be offset an equal amount to each side of Station 2 and will be parallel to the system impedance line (which lies to the right of Station 2), the right-hand characteristic shown on Fig. 23 will be nearer the system R-X plot. An examination of Fig. 23 leads us to select a setting of 1.25 ohms. A plot of the resultant characteristic is shown in Fig. 26. It will be noted that the relay characteristics on Fig. 26 have a definite margin to clear all system faults and yet provide extra operating margin by being well away from any stable operating area of the system.

If we investigate these settings in terms of their intersection with the "equal-excitation-swing line," as discussed previously, we find that "B" crosses at 168.5° and "A" crosses at 204°. The angle enclosed is therefore 35.5°. The fastest swing which will cross these characteristics in 0.005 second is one moving at 19.7 slip cycles per second. Thus, this setting provides more than adequate margin for the fastest swings anticipated. (It is interesting to note that the chart of Fig. 25b gives an answer in close agreement with that calculated above. Our relay setting of 1.25 ohms, expressed in per unit of the system ohms (15.92) would be 0.0785. S_{max} from Fig. 25b for this slip is between 19.5 and 20.0 slip cycles per second.)

This relay will operate only for an out-of-ste; condition and will provide a preselected system "split" point. The relay will operate on the firs swing cycle for velocities up to 19.7 slip cycles pe: second for all practical values of excitation.

Some speculation will arise as to the operation of this relay at abnormally small values of excita tion. Referring back to Fig. 14, it is mathemat ically possible, under the assumptions made ir constructing this diagram[4], for the E_A/E_B ratio to be reduced to a point where the out-of-step characteristic falls within the boundaries of the relay characteristics determined above. In the practical ease, this may or may not occur. Whether or not it does occur is a function of the slip as the machine enters this area of abnormally low excitation. If the slip is high, as would be expected for heavy initial loading of the generator, the apparent impedance will vary within a small area which may well be between the characteristics of the out-of-step relay. We are therefore forced to the conclusion that the out-of-step relay cannot reliably "double in brass" to provide protection in the event of abnormally small excitation. This brings us to a consideration of the loss-of-excitation protection problem.

[4]For numbered reference, see list at end of paper.

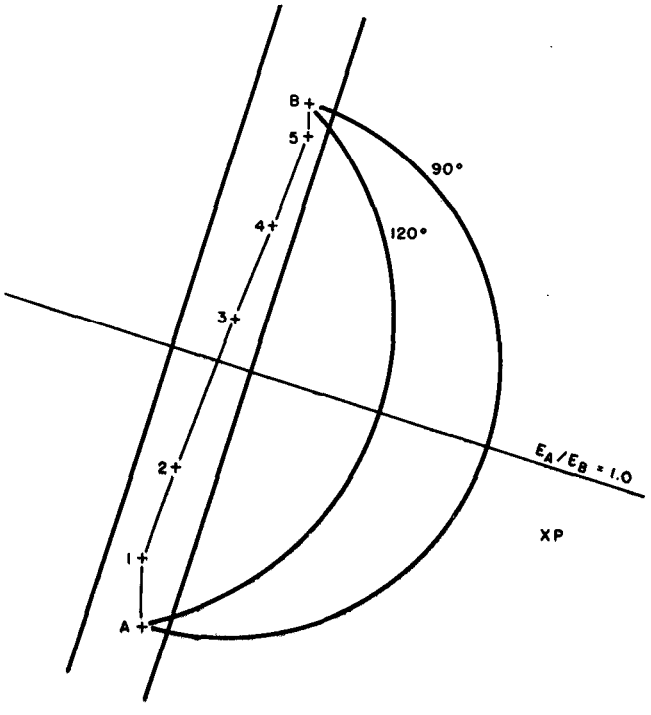


Fig. 26. Out-of-step Relay (set at 20%) Superimposed on Working Example, Fig. 17 (CEX Relay at Station 2)

LOSS-OF-EXCITATION CHARACTERISTICS

A failure in a machine's excitation system embraces a variety of abnormal field circuit conditions ranging from an effectively short-circuited field winding to an effectively open-circuited winding. The behavior of the machine between the time the excitation is affected and the time when the machine reaches a position nearly 180° out of phase with the system connected to the machine will vary, not only for each type of excitation failure, but also for each condition of initial machine loading. An R-X diagram of these loss-of-excitation characteristics will, however, resemble some combination of the two families of curves already studied. These are the curves of Fig. 14 and Fig. 15. Let us look at Fig. 15 first. These curves are plotted for a constant angular displacement between machines with varying excitation. Assuming the initial loading was such that Machine A was in advance of Machine B by 60 degrees, we would follow the 60° arc from some point (corresponding to the initial excitation) near the middle of the 60° arc in a clockwise fashion towards Point A as E_A decreased so that $E_A/E_B \rightarrow 0$. Actually, we would never reach Point A on the diagram, since the effective excitation will not reach zero because of induction. As we approach Point A then, our path must shift towards a constant excitation path as shown in Fig. 14. As the machine then advanced in phase angle with respect to Machine B, we would commence to trace one of the constant excitation circles about Point A in a counterclockwise direction.

The above speculative description is given merely as an introduction to this phenomena. It is not accurate, since the machines will not maintain exact angular relationships during the initial period of the curve, nor will they follow the constant excitation characteristic in the latter portions of the curve. A more basic reason back of this inaccuracy is that we have exceeded the assumptions used[4] to plot Fig. 14 and 15. During the initial part of the curve, the changes are relatively slow and are acting, not through the transient reactance, but through the equivalent reactance of the machines. This reactance, in turn, is a function of the excitation. Further, the machine tends to maintain constant power output, rather than constant angle during the curve. During the latter portion of the curve, the changes are acting through a reactance which will be changing from the direct to the quadrature-axis reactances. These reactances are in themselves a function of the slip of the machine. During the entire curve, the transition from equivalent reactance (steady-state loading) to transient reactance is a function of the speed at which the changes are taking place. This, in turn, depends to a large extent on the type of excitation failure and on the initial loading on the

machine. When we try to include all of these factors in our problem, we find that the only practical method for obtaining an analytical solution is by use of the differential analyzer.

From the differential analyzer, we have obtained four typical curves and sufficient additional data to enable us to proceed with our relay design. These curves are shown in Fig. 27. This shows these four characteristics plotted on an R-X diagram on a per-unit machine impedance base (e.g., the curve for rated machine armature current at rated terminal voltage would be a circle centered on the origin with a radius of 1.0/1.0 = 1.0 per unit impedance. For a more detailed discussion of this base, refer to Appendix A). (All of the subsequent R-X diagrams will be drawn to this base.) Each curve is marked at various points with reference numbers showing the time in seconds for the characteristic to reach that point from the initial loading point with the excitation system failure occurring at t = 0. Each curve has been terminated just before the machine first reaches the 180° position with respect to the machine back of the equivalent system impedance. This has been done only for the sake of clarity, since the curves do not end at these points, but the changes in impedance from then on become so erratic that the curve continuation becomes useless for our purpose.

With the help of a little imagination, it is not difficult to see that these curves are in general agreement with the speculative curve discussed previously. What is more important, in our problem of relay design, is that we have established a definite locus of the "end points" of these curves. This locus is shown by the heavy curve on Fig. 27, which represents the average of the direct- and quadrature-axis impedances about which the end of the loss-of-excitation characteristic varies. This average impedance is low at high slip, approaching the average of the direct-axis and quadrature-axis subtransient impedance of the generator. At very low values of slip, the characteristic approaches the average of the direct- and quadrature-axis synchronous impedances. These extremes are not actually reached, however.

We have established a curve which the loss-of-excitation characteristic must cross just before the affected machine passes 180° phase relationship with the system. Before we utilize this curve in designing our loss-of-excitation relay, we should investigate other abnormal, or normal, system operating characteristics which may approach this locus, and for which it may not be desirable for the loss-of-excitation relay to operate.

1. The first condition that comes to mind is the case of a fault out in the system for which operation of this relay would be undesirable. The system fault which approaches our loss-of-excitation locus most closely is a generator bus fault. If we visualize our loss-of-excitation relay as a single-phase relay (operated by line-to-line voltage and the vector difference of line currents), the apparent impedance viewed by the relay for all types of bus or system faults, lies in the region above the line MN[6] shown in Fig. 28. The line MN is perpendicular to the generator impedance AO. Arc

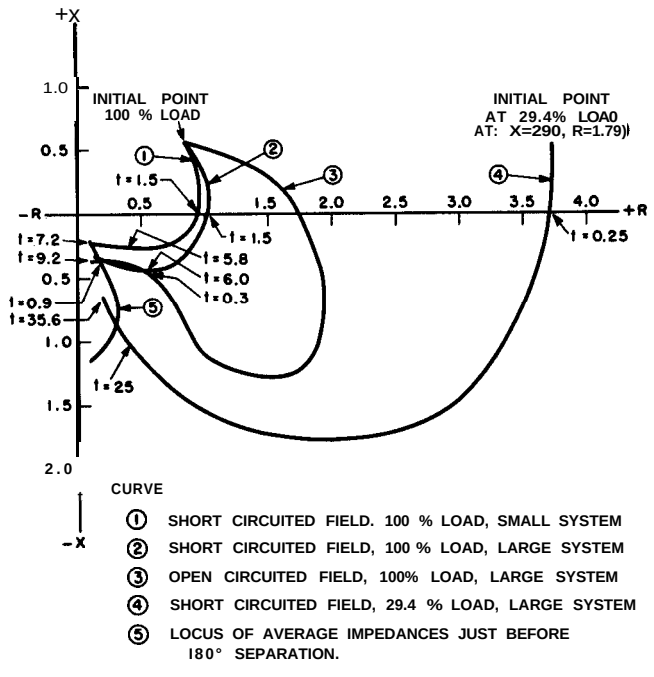


Fig. 27. Typical Loss-of-excitation *Curves* and Average "Locus"

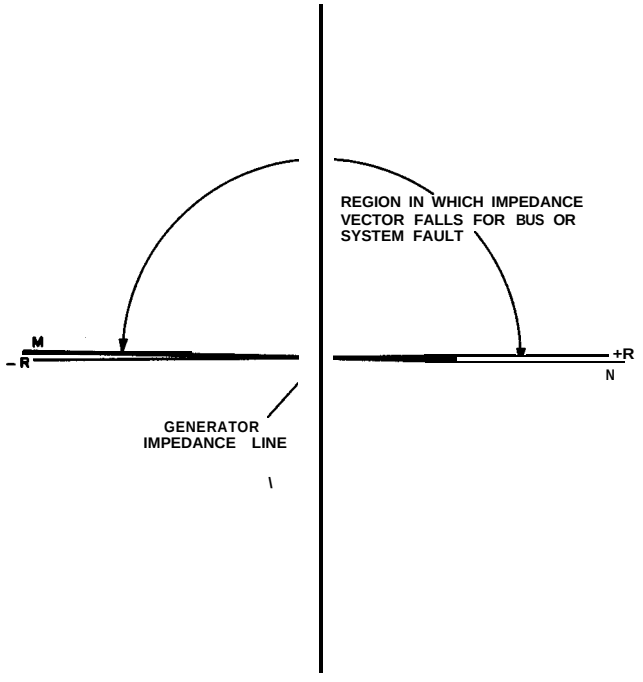


Fig. 28. Faults External to Generator

resistance may cause the apparent impedance to drop below the line MN[6], but this deviation is small and unimportant when we consider the next condition.

2. The second condition is an out-of-step condition between the generator and the system. The worst case, for our purpose, is that wherein the generator in question is tied directly to a system which is infinitely large compared to the generator rating. Since the $E_A/E_B = 1.0$ swing line is the perpendicular bisector of the total system impedance line (which is now reduced to the generator impedance because the system impedance is infinitely small), the "equal excitation swing" would follow the line shown in Fig. 29. Since this is a reasonable excitation condition, we may conclude that the loss-of-excitation relay characteristic should approach the origin no closer than one-half the generator impedance used in Fig. 29, i.e., one-half the transient impedance of the generator. We should remember that Fig. 29 represents an exaggerated case not likely to be closely approached in service. This is important because of the valid argument that a slight "normal" reduction in generator excitation would cause the out-of-step characteristic of Fig. 29 to move toward Point A (refer to Fig. 14). However, any increase in system impedance would cause these out-of-step characteristics to be shifted away from Point A. Since the loss-of-excitation relay should operate for impedances as low as the subtransient generator impedance, the limitation of one-half the transient impedance which we imposed above is certainly valid for general application.

(In connection with this discussion, we should realize that prolonged out-of-step operation of a generator may reduce the machine excitation even though the excitation source is unaffected. Under these conditions, however, operation of the loss-of-excitation relay would generally be considered acceptable, if not desirable.)

3. The third condition is that of purposefully reduced machine excitation, as dictated by system operating requirements. Since modern systems are just entering this area of system operation during light loading periods, it is advisable to review, briefly, a valuable reference on this subject. This is the Adams-McClure paper on "Underexcited Operation of Turbine Generators."[11] For simplicity, we have assumed throughout the following discussion that rated terminal voltage is maintained on the machine discussed. This assumption is of importance in converting from the KW-KVAR capability curve to the R-X diagram (for explanation of this point, see Appendix A.), and in evaluating statements made in this discussion. This

[6,11,12] For numbered reference, see list at end of paper.

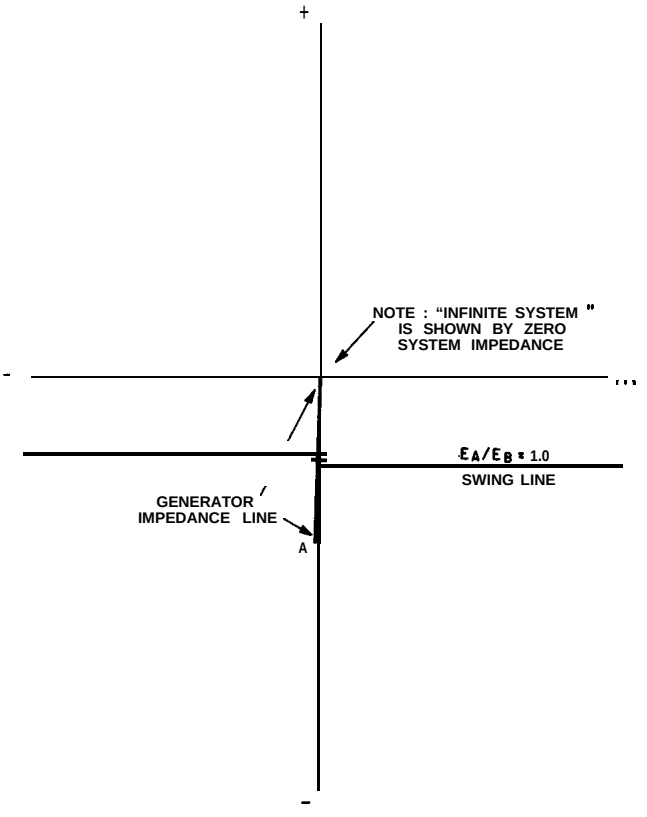


Fig. 29. **Out-of-step Characteristic-Generator lied Directly to Infinite System**

condition of maintaining rated machine terminal voltage can be realized, for steady-state conditions, through the use of a continuously acting voltage regulator. (The other assumptions made in the Adams-McClure paper[11] are more or less conventional and will not be repeated here.)

The pertinent information which we can glean from this paper can be shown as a plot of the generator operating limits in terms of KVAR and KW, with superimposed values of field current. This is shown in Fig. 30, with rated generator KVA as the base for per-unit notation of KW and KVAR, and "no load" field current as the base for per-unit notation of field current. This curve applies to a 0.8 p.f ., 0.9 S.C.R. machine. The portions of the curve can be readily identified as follows: Arc BC is the rated armature current limit, arc IB is the rated field current limit, and line GC represents the underexcited capability limitations of modern turbine-generators (as built by one manufacturer) as influenced by the tendency for heating in the end structures of the armatures[12]. Three steady-state-stability-limit curves are shown for further reference. These are curves : F-F' for the machine connected to an infinite system, F-D for the machine connected to a system five times as big as

the machine, and F-J for the machine connected to a system 2½ times as big as the machine.

Based on the obvious inferences of this diagram, the Adams-McClure paper[11] suggests a reactive lower-limit setting for the voltage regulator as shown by the shaded area on Fig. 30. That is, this reactive lower limit of the regulator governs the excitation so that the machine may not operate in those areas below the lower-limit settings.

Our next step is to show this suggested lower limit of regulated operation on the R-X diagram. This has been done in Fig. 31, where lines 1, 5 and 6 represent the lower-limit curves of Fig. 30. In order to **emphasize the reciprocal** relationship existing between Fig. 31 and Fig. 30, several other curves from Fig. 30 have been drawn on Fig. 31. The curves are identified as follows:

1. Rated armature current,
2. The P = 1.0 line,
3. The P = 0.5 line,
4. The constant field current = 0.5 curve,
5. The line forming the lower boundary, and
6. The line forming the upper boundary of the suggested reactive lower limit operation area.

Most of these curves are drawn only to help us interpolate from one drawing (Fig. 30) to the other (Fig. 31) and to emphasize the relationships existing on the R-X diagram. (Appendix A gives conversion equations for translating from the P-Q diagram to the R-X diagram.)

We will now proceed to draw only those curves affecting the desired operating characteristics of a loss-of-excitation relay on one R-X diagram in order to help us define that characteristic and apply a familiar relay element. Figure 32 shows:

[11] For numbered reference, see list at end of paper.

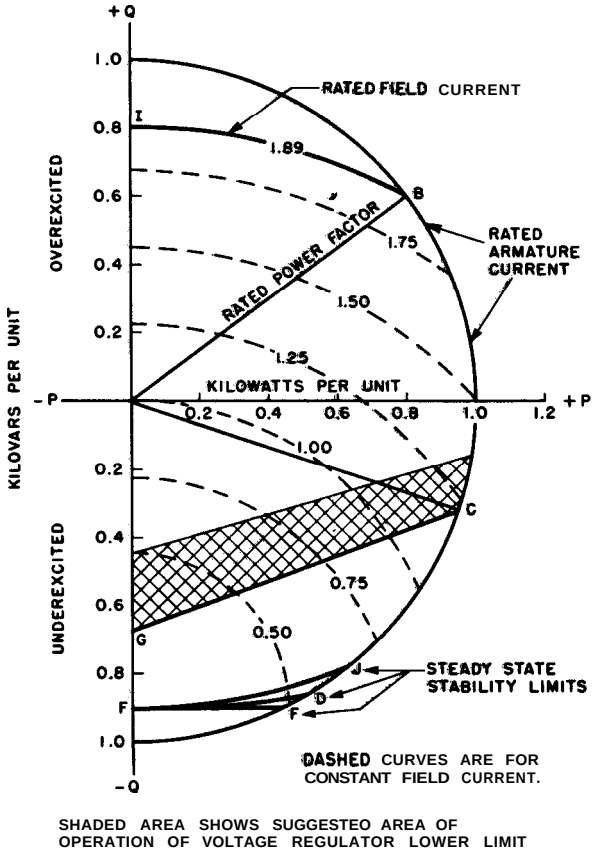


Fig. 30. **Rated Voltage Circle Diagram O.B., P.F., S.C.R. Cylindrical Rotor Generator**

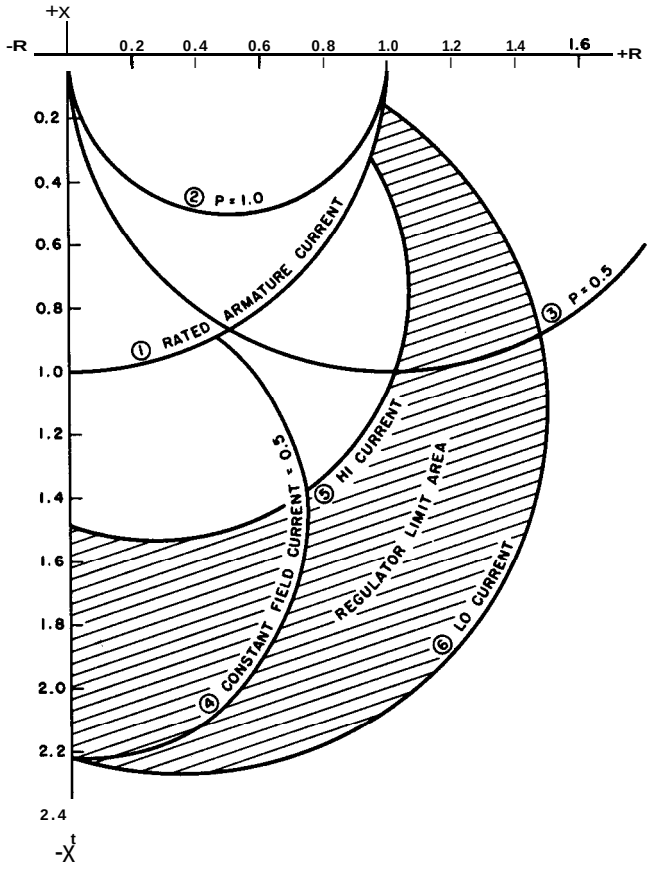


Fig. 31. Various *Generator* Circle Diagram Curves on an R-X Diagram

1. The locus of the end points of the loss-of-excitation characteristics,
2. An "equal voltage swing line" for the case of the generator connected to an infinite system,
3. The "high-current" boundary line for the suggested reactive lower-limit setting of a regulator.

Our job is to fit our loss-of-excitation relay characteristic about Curve 1 so that it will function for all types of excitation failures and yet keep this relay characteristic from trespassing on the "off-limits" areas suggested by the other curves. It is possible to visualize several combinations of ohmic relay characteristics which will do this job ; however, there is only one single-element ohmic relay characteristic that can be used by itself to meet all these requirements. This is an offset mho element having an angle of maximum torque of minus ninety degrees and an offset in the same direction. This characteristic is shown dotted on Fig. 32. Note that the offset has been made equal to one-half the transient impedance of the generator (in agreement with Curve 2 of Fig. 32) and that the diameter of the circle has been made equal to the synchronous impedance of the generator minus the amount of offset (so that the total "reach" of the relay equals the synchronous impedance of the generator). This setting enables the relay characteristic to be as small as possible and yet safely encompass an area into which all loss-of-excitation characteristics must enter before the machine passes the 180" point.

The area existing between Curve 3 (regulator reactive lower-limit curve) and the relay characteristic, shown in Fig. 32, prompts the observation that this relay may not function for certain cases of partial loss of excitation. Further study of Fig. 32 and Fig. 30 indicates that there is a region wherein this partial loss of excitation may be severe enough to exceed stability limits and to exceed desirable continuous machine operation capabilities, yet not cause relay operation. This is true, primarily because the relay setting was selected to provide as small a characteristic as possible consistent with positive operation. This was done because the relay is designed for high-speed tripping applications demanding that all precautions be taken against incorrect or undesirable tripping. In the case of such partial loss of excitation (i.e., causing a reduction of excitation beyond recommended machine operation, yet not severe enough

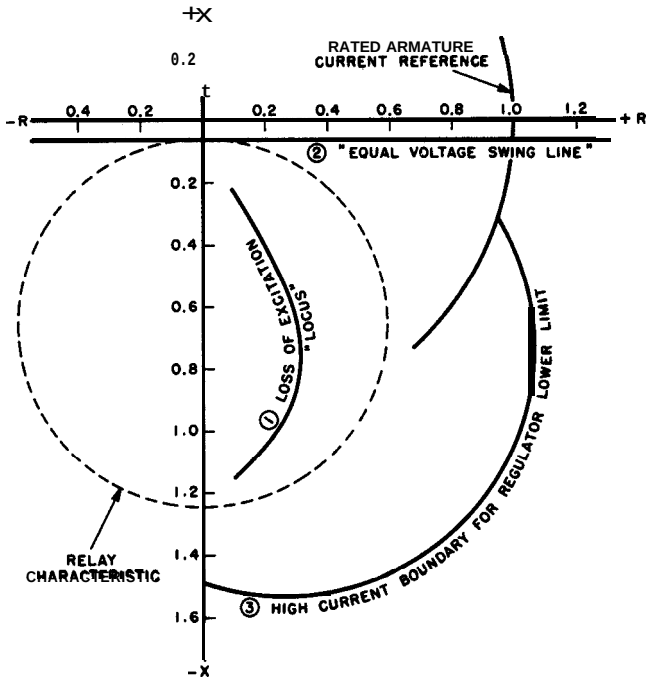


Fig. 32. Factors Involved in Loss-of-excitation Protection

to operate the relay), it is entirely likely that immediate tripping of the machine would not be desirable. For example, the operator would have more time in which to act and could possibly avoid the necessity for tripping the machine under this condition. It would be desirable to provide a separate, more sensitively set relay to sound an alarm and possibly to trip after a long time delay in this case, particularly when a reliable means for tripping in the event of a progressive excitation failure is in service.

The above discussion is meaningless in the event this partial loss of excitation involved factors requiring immediate action such as a short circuiting of part of the generator field poles, which could cause serious vibration. Protection against this type of failure should more properly be left to vibration-detecting means or field ground detectors.

Further study of Fig. 32 leads to the observation that the loss-of-excitation relay may operate for a fault within the generator which it protects. This should cause little concern one way or the other, since the generator must be disconnected from the system anyway.

APPENDIX A

PER-UNIT NOTATION

The calculation of machine or system performance may be simplified by the use of per-unit representation of all quantities such as voltage, current, impedance, power, or KVA. Thus, a selected base value is considered as unit, or 1.0, and all quantities expressed as a ratio in decimal form with respect to the base value of the quantity. For convenience, it has been the practice to select a common base KVA and use with the rated line-to-line voltage as the independent base quantities. The relationship of other base quantities is determined by these equations :

$$\text{Base Amps} = \frac{\text{Base KVA}}{\sqrt{3}\ \text{Base KV}}$$

$$\text{Base Ohms} = \frac{\text{L-G KV} \times 10^3}{\text{Base Amps}} = \frac{\text{Base KV} \times 10^3}{\sqrt{3}\ \text{Base Amps}} = \frac{\text{Base KV}^2 \times 10^3}{\text{Base KVA}}$$

$$\text{Base KW} = \text{Base KVAR} = \text{Base KVA}$$

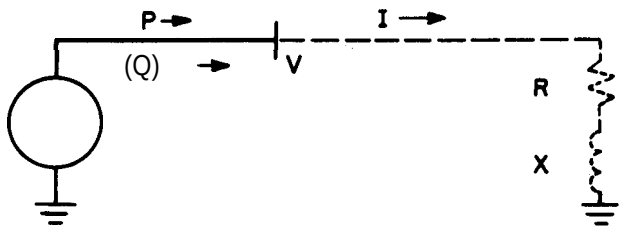
and so forth.

For example, a 60,000-KW preferred standard turbine-generator operating at the 110 percent turbine rating, at rated 0.85 p.f., will have an output of 66,000 KW and 40,900 KVAR, or 77,600 KVA. For a machine operating at its rated voltage, assuming that voltage to be 14 KV, the current flowing would be 2720 – j1690 or 3200 amperes. The series load impedance causing this flow would be 2.15 ohms resistance and 1.33 ohms reactance.

To express these quantities in per unit based on the machine's ½ psi H_2 rating and rated voltage : Base KVA = 70,600 KVA, Base KV = 14 KV, Base Amperes = 2910 amps, Base ohms = 2.78 ohms. Converting the flow figures determined above to per unit: P = 0.935, Q = 0.580, KVA = 1.1, I = 0.935 – j0.580 = 1.1 and the series load impedance is 0.77 per-unit resistance and 0.48 per-unit reactance.

Where per-unit quantities are used throughout, the conversion becomes more straightforward.

Let us represent the load flow by the following diagram :



The following relationships hold :

$$I = \frac{\sqrt{P^2 + Q^2}}{V} \text{ and } I^2 = \frac{P^2 + Q^2}{V^2}$$

$$R = \frac{P}{I^2} = \frac{PV^2}{P^2 + Q^2} \text{ and } X = \frac{Q}{I^2} = \frac{QV^2}{P^2 + Q^2}$$

In the above example then:

$$R = \frac{0.935 \times (1.0)^2}{(1.1)^2} = 0.772$$

$$X = \frac{0.580 \times (1.0)^2}{(1.1)^2 \times 1.0} = 0.480$$

Conversion from per-unit diagrams, i.e., P-Q to R-X or vice versa, can be easily accomplished by the following equations :

$$R = \frac{PV^2}{P^2 + Q^2} \qquad P = \frac{RV^2}{R^2 + X^2}$$

$$X = \frac{QV^2}{P^2 + Q^2} \qquad Q = \frac{XV^2}{R^2 + X^2}$$

These equations may also be used for actual values, in which case it is imperative that consistent units be used.

Thus :

R = line-to-neutral resistance in ohms
X = line-to-neutral reactance in ohms
V = line-to-line voltage in volts
P = 3-phase power, supplied by generator to system, in watts
Q = 3-phase vars, supplied by "overexcited" generator to system for positive values, in volt-amperes.

(The above are all positive phase sequence values.)

CONCLUSION

We have seen that the R-X diagram is an essential tool in the comparison, evaluation, and application of distance relays. (It is also essential in numerous other fields not covered in this discussion.) The diagram is easy to construct and lends itself to a simultaneous plot of system conditions and relay characteristics with a simplicity of geometric constructions. By its simplicity, it offers a readily understandable picture of complex relationships in the complex field of modern relaying.

This has been a hurried and limited trip through the features of the R-X diagram as applied to relays. Many simplifying assumptions have been made and many interesting relay applications have been avoided. For those of you who are interested in further exploration of the possibilities of the R-X diagram, we recommend, most wholeheartedly, Miss Clarke's excellent paper on this subject. Full reference information follows:

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